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Bayesian Single Sampling Attribute Plans

for Discrete Prior Distributions.

By

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Institute of Mathematical Statistics, University of Copenhagen.
Technical Report No. 5 to the Office of Naval Research
prepared under Contract Nonr-N62558-3073 (NR 042-228).

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June 1964.

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1. Introduction and summary.

The main purpose of the present paper is to give a rather complete tabulation and discussion of properties of a system of single sampling attribute plans obtained by minimizing average costs under the assumptions that costs are linear in p, the fraction defective, and that the distribution of lot quality is a double binomial distribution.

Starting from a cost function containing 6 parameters and a mixed binomial prior distribution it is shown how the average costs may be written in a standard form containing only two parameters, p_r and p_s, besides the parameters defining the prior distribution. The one parameter, p_r, is the economic break-even quality and depends on the costs of acceptance and rejection only, whereas the second parameter, p_s, also depends on the costs of sampling inspection and the average quality. In a simple and practically important case p_r and p_s denote the costs of rejection and the costs of sampling inspection, respectively, divided by the costs of accepting a defective item.

Specializing the prior distribution to a double binomial distribution defined by the two quality levels (p_1,p_2) and the weights (w_1,w_2), $w_1+w_2=1$, it will be seen that the optimum sampling plan (n,c) depends on the 6 parameters (N,p_r,p_s,p_1,p_2,w_2) where N denotes lot size. It may be shown, however, that the weights combine with the p's in such a way that only 5 (independent) parameters are left.

A procedure to obtain the exact solution of the problem has been developed in a previous paper and this has been used for computing a set of master tables in which $p_r = p_s = 0.01$ and 0.10, $w_2 = 0.05$, (p_1,p_2) take on suitably chosen values in relation to the value of p_r, and $1 \le N \le 200{,}000$.

Prepared with the partial support of the Office of Naval Research (Nonr-N62558-3073).

In the remaining part of the paper the properties of the optimum plans are studied with the purpose to derive simple conversion formulas which will make it possible to find the optimum plan for an arbitrary set of parameters from a plan in the master table with a "corresponding" set of parameters. The main tool for this investigation is the asymptotic expressions for the acceptance number and for the sample size, viz.

$$c = np_o + a + o(1) \text{ and } n = \frac{1}{\varphi_o}(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_o) + o(1),$$

where p_o and φ_o are functions of (p_1,p_2) only, whereas a and λ depend on the other parameters also. It is furthermore shown that the minimum value of the standardized costs per lot asymtotically equals the costs of sampling inspection plus a constant (depending on (p_1,p_2)) and that the producer's and the consumer's risks tend to zero inversely proportional to lot size. Numerical investigations show that the asymptotic expressions give good approximations to the optimum plan even for quite small values of N.

By means of the asymptotic formulas it is possible to find out how (n,c) vary with the individual parameters. One of the most important results is found by letting all the p's tend to zero which leads to "the proportionality law": The optimum sampling plan corresponding to $(N,\lambda p_r,\lambda p_s,\lambda p_1,\lambda p_2,w_2)$ is approximately equal to $(n^*/\lambda,c^*)$ where (n^*,c^*) is the plan corresponding to $(N^*,p_r,p_s,p_1,p_2,w_2)$ with $N^*= N\lambda$.

This theorem combined with other similar results regarding the effect of varying the individual parameters lead to two general conversion formulas stated in sections 8 and 11. A summary of these formulas is given at the end of the paper in connection with the tables.

Efficiency of a sampling plan is defined as the ratio of the standardized costs (loss) of the optimum plan and the costs of the plan in question. Efficiency is discussed for various alternative systems and the efficiency of using optimum plans determined from wrong values of the parameters is studied.

Finally the present system is discussed in relation to other systems and it is pointed out that from an economic point of view it is not advisable to fix the consumer's or the producer's risk. If one wants a system with a fixed risk then the risk should be fixed to 50 per cent at a point between p_1 and p_2. Two such IQL systems are then briefly discussed.

2. The model.

Several authors have studies economic models, mostly linear, for the determination of single sampling inspection plans by attributes, see for instance [1] and [2].

We shall here start from the formulation proposed by Guthrie and Johns [3] and show how the model may be reduced to a standard form as previously used by Hald [4].

Let N and n denote lot size and sample size and let X and x denote number of defectives in the lot and the sample, respectively. The acceptance number is denoted by c.

Let the costs be

$$nS_1 + xS_2 + (N-n)A_1 + (X-x)A_2 \quad \text{for } x \leq c \qquad (1)$$

and

$$nS_1 + xS_2 + (N-n)R_1 + (X-x)R_2 \quad \text{for } x > c. \qquad (2)$$

The interpretation of the six cost parameters depends on the kind of inspection envisaged, i.e. whether inspection is a consumers receiving inspection, a producers inspection of finished goods, or "internal inspection" by delivery of goods from one department to another within the same firm. The cost parameters may have quite different values when considered exclusively froma producer's or a consumer's point of view because certain costs are borne primarily by one of the parties involved. The values of the cost parameters also depend on whether the inspection is rectifying or non-rectifying, destructive or non-destructive. In the following the two cost expressions are discussed and a few examples of interpretation are given.

Costs associated with the sample, $nS_1 + xS_2$, for brevity called "costs of sampling inspection" consist of two parts: one part, nS_1, proportional to the number of items in the sample so that S_1 includes sampling and testing costs per item, and another part, xS_2, proportional to the number of defectives in the sample, i.e. S_2 denotes additional costs for an inspected defective item. If defective items found in the sample are repaired, say, then S_2 may include the repair costs per item.

"Costs of acceptance" are similarly composed of a part, $(N-n)A_1$, proportional to the number of items in the remainder of the lot, and another part, $(X-x)A_2$, proportional to the number of defective items accepted. Whereas A_1 usually will be zero or negligible, A_2 will often be considerable. If accepted items are used as parts in an assembly operation, say, A_2 may include the manufacturing costs (or the price) of an item, the costs of handling the defective item in assembling and disassembling, and the damage done to other parts used in the assembly. In case of inspection of finished goods A_2 may include costs of repair, service and guarantees plus loss of good-will.

"Costs of rejection" consist of a part, $(N-n)R_1$ proportional to the number of items in the remainder of the lot, and another part, $(X-x)R_2$, proportional to the number of defective items rejected. Rejection is here taken in a broad sense meaning only that

the lot cannot be accepted according to the sampling plan used. Rejection may therefore lead to sorting, price reduction, scrapping, or salvaging. If rejection means sorting, say, then R_1 includes sorting costs per item and R_2 denotes additional costs for defective items found, for example costs of repair or replacement.

It is obvious that from a practical point of view it will in general be easiest to obtain information on the values of the cost parameters in the case of "internal inspection".

Denoting the hypergeometric probability by

$$p(x|X) = \binom{n}{x}\binom{N-n}{X-x}/\binom{N}{X}$$

the average costs for lots of size N with X defectives become

$$K(N,n,c,X) = \sum_{x=0}^{n} (nS_1 + xS_2)p(x|X) + \sum_{x=0}^{c} ((N-n)A_1+(X-x)A_2)\,p(x|X) + \sum_{x=c+1}^{n} ((N-n)R_1+(X-x)R_2)\,p(x|X). \tag{3}$$

Let $f_N(X)$ denote the (prior) distribution of X, i.e. the distribution of lot quality. The average costs then become

$$K(N,n,c) = \sum_{X} K(N,n,c,X)\,f_N(X). \tag{4}$$

As shown in [4] this expression becomes linear in N for the important class of mixed binomial distributions, i.e. for

$$f_N(X) = \int_0^1 \binom{N}{X} p^X q^{N-X}\, dW(p) \tag{5}$$

where W(p) denotes a cumulative distribution function (independent of N).

From (3) - (5) we find

$$K(N,n,c) = \int_0^1 K(N,n,c,p)\,dW(p) \tag{6}$$

where

$$K(N,n,c,p) = n(S_1 + S_2p) + (N-n)((A_1 + A_2p)P(p) + (R_1 + R_2p)Q(p)) , \tag{7}$$

$$P(p) = B(c,n,p) = \sum_{x=0}^{c} \binom{n}{x} p^x q^{n-x}, \tag{8}$$

and Q(p) = 1-P(p).

For convenience the frequency function corresponding to W(p) will be called the distribution of the process average or the distribution of p as distinct from $f_N(X)$

where

which gives the distribution of X/N, i.e. the distribution of lot quality. (The following discussion will be in terms of p).

Limiting the prior distributions to mixed binomials (6) shows that the average costs may be considered as an average of the cost function (7), which is a function of p, with respect to the distribution of p. It should be noted that this result is valid for any (N,n) for a mixed binomial prior distribution and that a similar result holds for $N \to \infty$, $n \to \infty$, and $n/N \to 0$, for any prior distribution. The limit theorems derived in the following may therefore be applied in general.

The sampling plans discussed are obtained by minimizing $K(N,n,c)$ according to (6) with respect to (n,c) for given cost parameters and prior distribution and they will be called Bayesian single sampling plans or optimum plans.

Starting from (7) we introduce the three cost functions

$$k_s(p) = S_1 + S_2 p, \tag{9}$$

$$k_a(p) = A_1 + A_2 p, \tag{10}$$

and

$$k_r(p) = R_1 + R_2 p, \tag{11}$$

defined for $0 \leq p \leq 1$. We shall make the following assumptions regarding these functions:

1. All three functions are non-negative and none of them is identical zero.

2. $k_a(0) < k_r(0)$ and $k_a(1) > k_r(1)$, from which follows that the equation $k_a(p)=k_r(p)$ has the solution

$$p_r = (R_1 - A_1)/(A_2 - R_2), \quad 0 < p_r < 1, \tag{12}$$

p_r being called the (economic) break-even quality.

3. $k_s(p) \geq k_m(p)$ for $0 \leq p \leq 1$, where

$$k_m(p) = \begin{cases} k_a(p) & \text{for } p \leq p_r \\ k_r(p) & \text{for } p > p_r . \end{cases} \tag{13}$$

The function $k_m(p)$ gives the unavoidable (minimum) costs, i.e. the costs corresponding to the situation where perfect knowledge of quality exists without costs and all lots (processes) are classified correctly on that basis, viz. accepted for $p \leq p_r$ and rejected for $p > p_r$.

Averages over the prior distribution are denoted by k_s, k_a, etc., i.e.

$$k_a = \int_0^1 k_a(p)dW(p) = k_a(\bar{p}) = A_1 + A_2\bar{p}, \tag{14}$$

and

$$k_m = \int_0^1 k_m(p)dW(p) = \int_0^{p_r} k_a(p)dW(p) + \int_{p_r}^1 k_r(p)dW(p). \tag{15}$$

Costs per <u>item</u> are denoted by k, costs per <u>lot</u> by the corresponding K, i.e. $K = Nk$.

The average costs for the <u>three cases without sampling inspection</u>, i.e. the cases where

(a) all lots are classified correctly,

(b) all lots are accepted, and

(c) all lots are rejected,

then become k_m, k_a, and k_r, respectively. These cases are useful "reference cases" since sampling inspection is justified only if $k - k_m < \min\{k_a - k_m, k_r - k_m\}$, where $k = K(N,n,c)/N$.

Case (a) will usually be considered as the basic reference case and average costs for other cases will therefore be reduced by k_m, since k_m represents the average fixed costs per item which will be incurred irrespective of the decision made. The <u>cost differences</u>

$$k_a - k_m = \int_{p_r}^1 (k_a(p) - k_r(p))dW(p)$$

and

$$k_r - k_m = \int_0^{p_r} (k_r(p) - k_a(p))dW(p)$$

represent <u>average decision losses</u> in case (b) and (c) respectively, and $k_s - k_m$ represents the average "loss" by inspection.

From (6) and (15) we find

$$K = nk_s + (N-n)\int_0^1 (k_a(p)P(p) + k_r(p)Q(p))dW(p) \tag{16}$$

and

$$K_m = nk_m + (N-n)\{\int_0^{p_r} k_a(p)dW(p) + \int_{p_r}^1 k_r(p)dW(p)\}$$

leading to

$$K - K_m = n(k_s - k_m) + (N-n)\{\int_0^{p_r} (k_r(p)-k_a(p))Q(p)dW(p) + \int_{p_r}^1 (k_a(p)-k_r(p))P(p)dW(p)\}$$

$$= n(k_s - k_m) + (N-n)(A_2 - R_2)\{\int_0^{p_r} (p_r - p)Q(p)dW(p) + \int_{p_r}^1 (p - p_r)P(p)dW(p)\}, \tag{17}$$

the two terms giving the average costs of sampling inspection and the average

Instead of minimizing K with respect to (n,c) we might just as well minimize $K-K_m$, $(K-K_m)/(A_2-R_2)$, or $(K-K_m)/(k_s-k_m)$, since K_m, A_2-R_2, and k_s-k_m are independent of (n,c). It will be seen from (17) that it is practical to use A_2-R_2 or k_s-k_m as "economic unit".

Defining

$$p_m = \int_0^{p_r} p\,dW(p) + \int_{p_r}^{1} p_r\,dW(p) = p_r - \int_0^{p_r} (p_r-p)\,dW(p), \tag{18}$$

we find $0 \leq p_m \leq p_r$ and

$$p_r - p_m = (k_r - k_m)/(A_2 - R_2). \tag{19}$$

Defining p_s by means of

$$p_s - p_m = (k_s - k_m)/(A_2 - R_2) \tag{20}$$

we find

$$p_s - p_r = (k_s - k_r)/(A_2 - R_2) \tag{21}$$

and

$$p_s = \{(S_1 - A_1) + (S_2 - R_2)\bar{p}\}/(A_2 - R_2). \tag{22}$$

Introducing

$$R^*(N,n,c) = \{K(N,n,c) - K_m\}/(A_2 - R_2) \tag{23}$$

we find the <u>standard form</u>

$$R^* = n(p_s - p_m) + (N - n)\{\int_0^{p_r} (p_r - p)Q(p)\,dW(p) + \int_{p_r}^{1} (p - p_r)P(p)\,dW(p)\} \tag{24}$$

containing only two parameters p_r ans $p_s - p_m$, instead of the six cost parameters in the original model, see [4]. It should be noted that $p_s - p_m$ depends on the prior distribution besides on the cost parameters.

Consider the <u>special case</u> given by $k_a(p) = A_2p$, $k_r(p) = R_1$, and $k_s(p) = S_1$, which is a model commonly used in practice. It follows that $p_r = R_1/A_2$ and $p_s = S_1/A_2$, i.e. p_r and p_s are the costs of rejection and of sampling and testing, respectively, measured with the cost of accepting a defective item. This simple interpretation of p_r and p_s is one of the reasons for using them as parameters.

It is often useful to discuss the problem in terms of the simple cost functions $k_a(p) = p$, $k_r(p) = p_r$, and $k_s(p) = p_s$, which immediately lead to the form (24). The corresponding form of (7) becomes

$$K_o(p) = np_s + (N - n)(pP(p) + p_rQ(p))$$

from which the general form may be found as

$$K(p) = (A_2 - R_2)K_o(p) + (nS_2 + (N - n)R_2)p + (NA_1 - n(S_2 - R_2)\bar{p}).$$

A sketch of the cost functions for a typical case has been given in Fig. 1, which is based on the data in section 13.

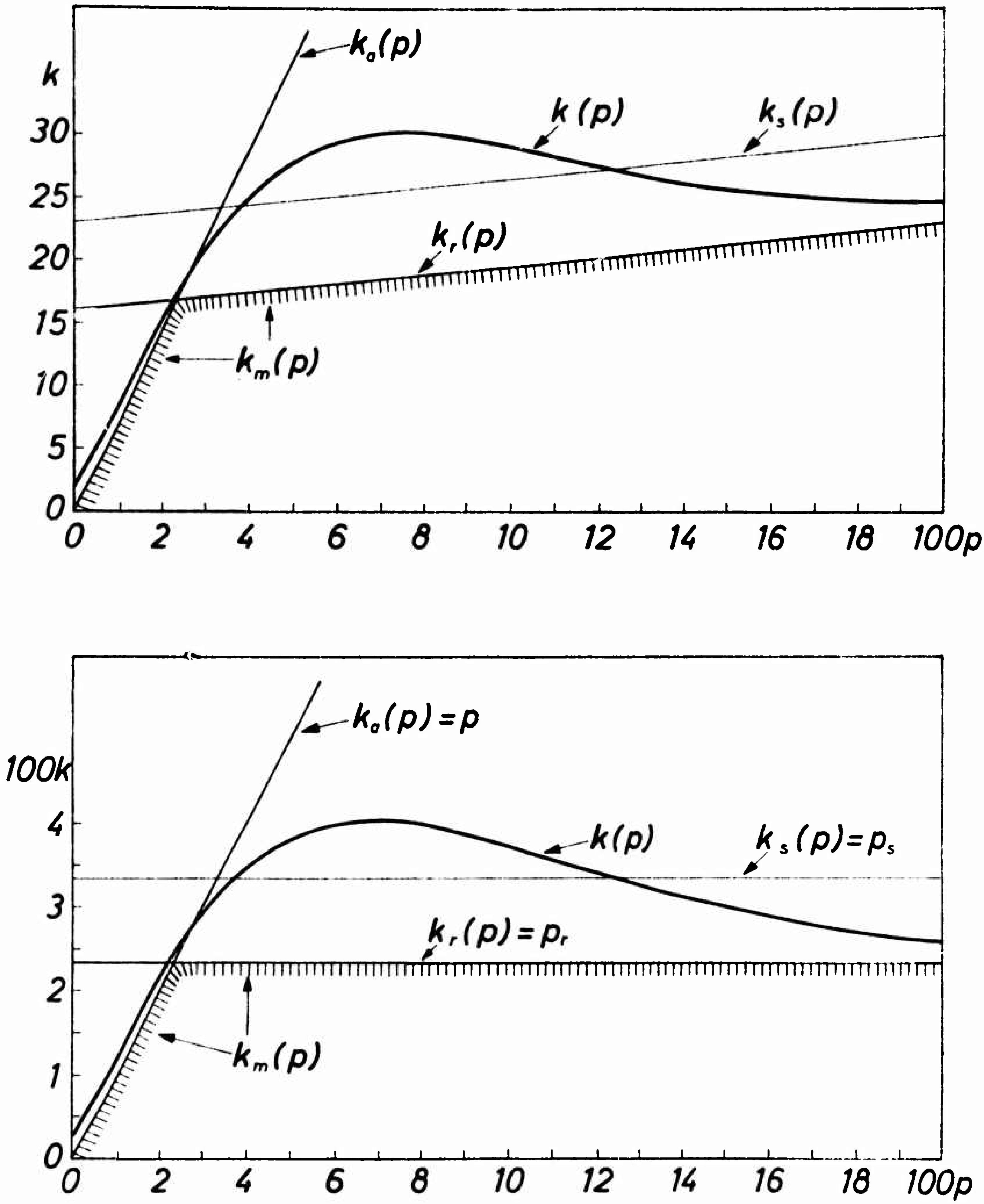

Fig. 1

Example of cost functions.

For some purposes it is useful to use $k_s - k_m$ as economic unit instead of $A_2 - R_2$. Putting

$$R(N,n,c) = \{K(N,n,c) - K_m\}/(k_s - k_m),$$

i.e.

$$R = R^*/(p_s - p_m),$$

we find

$$R = n + \frac{N-n}{p_s - p_m}\left\{\int_0^{p_r}(p_r - p)Q(p)dW(p) + \int_{p_r}^{1}(p - p_r)P(p)dW(p)\right\}, \tag{25}$$

the two terms again giving the costs of sampling inspection and the average decision losses, respectively, but here using the average costs of sampling inspection (minus k_m) per item in the sample as economic unit.

In the next section we shall discuss the determination of (n,c) for a <u>double binomial distribution</u> as prior distribution. This means that p is a random variable taking on only two values, $p_1 < p_r < p_2$, with probabilities w_1 and $w_2 = 1 - w_1$, respectively. From (25) we then find

$$R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2)) \tag{26}$$

where

$$\gamma_i = |p_i - p_r| w_i/(p_s - p_m) = |k_a(p_i) - k_r(p_i)| w_i/(k_s - k_m),\ i=1,2, \tag{27}$$

$$p_m = p_1 w_1 + p_r w_2\ , \tag{28}$$

i.e. R depends on four parameters only, viz. $p_1, p_2, \gamma_1, \gamma_2$.

The correspondingly standardized costs for the cases of acceptance and rejection without inspection are

$$R_a = N(k_a - k_m)/(k_s - k_m) = N\gamma_2 \tag{29}$$

and

$$R_r = N(k_r - k_m)/(k_s - k_m) = N\gamma_1. \tag{30}$$

These results may also be obtained from (26) for $n = 0$ by setting $P(p) = 1$ and 0, respectively.

If acceptance without inspection is cheaper than rejection without inspection, i.e. $k_a < k_r$ we find $\bar{p} < p_r$ and $\gamma_2 < \gamma_1$.

In the special case $k_s = k_r$ we have $p_s = p_r$ and $\gamma_1 = 1$ so that the model contains only three parameters.

It should be noted that

$$\gamma_2 = \frac{\bar{p} - p_m}{p_s - p_m} = 1 - \frac{p_s - \bar{p}}{p_s - p_m} \tag{31}$$

and

$$\gamma_1 = \frac{p_r - p_m}{p_s - p_m} = 1 - \frac{p_s - p_r}{p_s - p_m} . \tag{32}$$

3. The exact solution and the tables.

In a previous paper [4] we have proved the following theorem:

For a double binomial (prior) distribution of lot quality given by the parameters (p_1, p_2, w_2) and for linear cost functions (1) and (2) the Bayesian single sampling plan may be found by minimizing R (N,n,c), see (26), with respect to (n,c). The solution satisfies the two inequalities

$$\alpha + \beta c \leq n < \alpha + \beta(c + 1) \tag{33}$$

and

$$F(n - 1,c) \leq N < F(n,c) \tag{34}$$

where

$$\alpha = \log \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)} \Big/ \log \frac{q_1}{q_2} = \log \frac{\gamma_2}{\gamma_1} \Big/ \log \frac{q_1}{q_2} , \tag{35}$$

$$\beta = \log \frac{p_2 q_1}{q_2 p_1} \Big/ \log \frac{q_1}{q_2} , \tag{36}$$

and

$$F(n,c) = n + 1 + \frac{p_s - p_r + \sum_i w_i(p_r - p_i)B(c,n,p_i)}{\sum_i w_i(p_i - p_r)p_i b(c,n,p_i)} . \tag{37}$$

For two plans (n_1,c_1) and (n_2,c_2) , $c_1 < c_2$ say, satisfying (33) and having overlapping N-intervals according to (34) $R(N,n_1,c_1) \lesseqgtr R(N,n_2,c_2)$ for $N \lesseqgtr N_{12}$ where

$$N_{12} = \frac{(p_s - p_r)(n_2 - n_1) + n_2\gamma(n_2,c_2) - n_1\gamma(n_1,c_1)}{\gamma(n_2,c_2) - \gamma(n_1,c_1)} \tag{38}$$

and

$$\gamma(n,c) = \sum_i w_i(p_r - p_i)B(c,n,p_i). \tag{39}$$

In [4] the theorem was derived as a special case of a more general one. We shall here derive the theorem directly from (26) using the same method as in [4].

Values of (n,c) minimizing R must satisfy the two inequalities

$$\Delta_c R(N,c-1,n) \leq 0 < \Delta_c R(N,c,n), \qquad 0 \leq c \leq n, \tag{40}$$

and

$$\Delta_n R(N,c,n-1) \leq 0 < \Delta_n R(N,c,n), \qquad c \leq n \leq N, \tag{41}$$

Δ denoting the usual forward difference operator.

Noting that $\Delta_c B(c,n,p) = b(c+1,n,p)$ and $\Delta_n B(c,n,p) = -pb(c,n,p)$ we find from (26)

$$\Delta_c R(N,n,c) = (N-n)\,\{-\gamma_1 b(c+1,n,p_1) + \gamma_2 b(c+1,n,p_2)\} \tag{42}$$

and

$$\Delta_n R(N,n,c) = 1-\{\gamma_1 Q(p_1)+\gamma_2 P(p_2)\} + (N-n-1)\{\gamma_1 p_1 b(c,n,p_1)-\gamma_2 p_2 b(c,n,p_2)\} \tag{43}$$

Inserting these expressions into (40) and (41) and solving for n and N, respectively, immediately leads to (33) and (34). From $R(N,n_1,c_1) = R(N,n_2,c_2)$ we next determine N_{12} by solving for N.

A sketch of R as function of n and c for fixed N has been given in Fig. 2 for a typical case.

The economic interpretation of (40) and (42) is the following: For given n the optimum value of c is determined such that a change of c, an increase by 1 say, will give nearly no change of the total decision loss, since the loss due to the increased consumer's risk is nearly balanced by the gain due to the smaller producer's risk.

Similarly the interpretation of (41) and (43) is that for given c the optimum value of n is determined such that a change of n, an increase by 1 say, will give nearly no change of total costs, since the increase of sampling inspection costs by 1 minus the average decision loss for one item is nearly balanced by the decrease in decision losses for the remainder of the lot.

Tabulation of optimum plans may be carried out by starting from the smallest value of c giving a positive $n (n \geq c)$ according to (33), i.e. $c_m = [-\alpha/(\beta-1)]$, [ ] denoting "the integer part of". For consecutive values of c, n- and N-intervals are computed from (33) and (34) and in case of overlapping N-intervals costs are compared by means of (38). A detailed example may be found in [4]. The tables have been computed by this method on an electronic computer.

The sampling plans have been tabulated for two "quality levels", viz. $p_r = p_s = 0.01$ and 0.10, for one value of the weight function $w_2 = 0.05$, for 8 values of p_1/p_r, and for 10, respectively 5, values of p_2/p_r, giving a total of 120 tables. Each table gives (n,c) as function of N for $N \leq 200{,}000$.

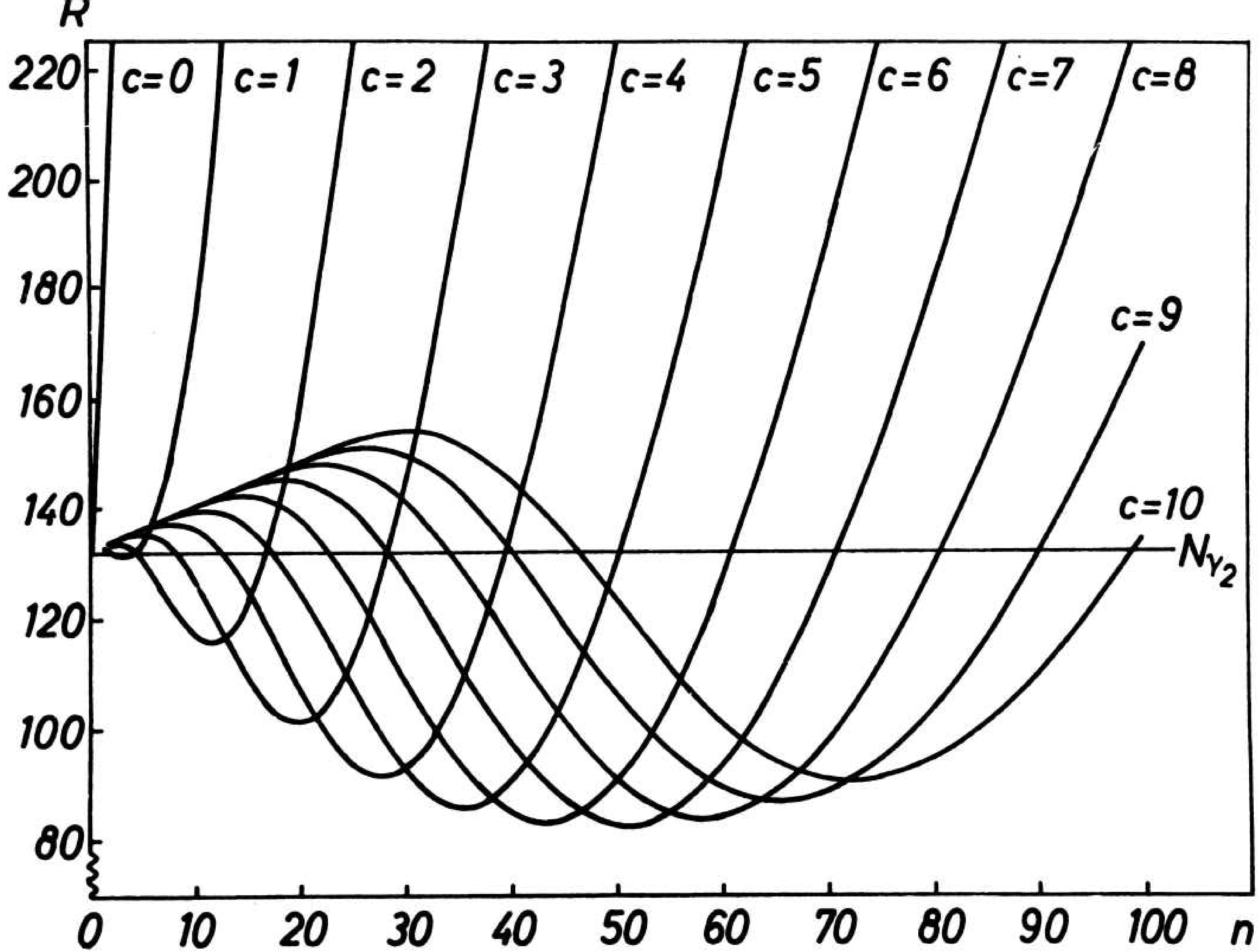

Fig. 2

R(N,n,c) as function of n and c for N = 1000, $P_r = P_s = 0.10$, $P_1 = 0.06$, $P_2 = 0.20$, and $w_2 = 0.05$.

For $p_r = 0.01$ the search for optimum plans has been limited to values of n which are multiples of 5.

These tables will be referred to as "master tables" since optimum plans for other values of the parameters may easily be found from the tabulated ones by means of conversion formulas developed in the following sections.

The exact solution has been modified in one respect. For a given value of c the first and last N-interval may be rather short as compared to the other intervals. As an example consider the following section of the original table for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.020$, $w_2 = 0.05$:

N	n	c	ΔN
4010 - 4370	165	3	360
4370 - 4420	170	3	50
4420 - 4430	240	4	10
4430 - 4920	245	4	490
4920 - 5570	250	4	650
5570 - 5590	255	4	20
5590 - 5610	325	5	20
5610 - 6250	330	5	640

The example shown is an extreme one with small intervals occurring at the beginning as well as at the end of each section of the table. It is naturally without any interest to use the sampling plan (240,4) for $4420 < N < 4430$ and then change to (245,4) for $4430 < N < 4920$. To eliminate such small intervals from the final table it was decided to discard the first and the last sampling plan for a given c if the length of the corresponding N-interval was less than 1/5 of the length of the neighbouring interval. In such cases the value of N according to (38) was computed for the new neighbouring plans, (165,3) and (245,4) say, to find the optimum N-intervals for the remaining plans. The result of this procedure is in most cases practically equal to incorporating the small N-intervals into the larger neighbouring intervals, for example using (245,4) for $4420 < N < 4920$.

To save space every second N-interval for a given value of c has been omitted because the corresponding sampling plans may be found by adding 1 ($p_r = 0.10$) and 5 ($p_r = 0.01$), respectively, to n for the preceding interval.

Values of N have been rounded to 3 significant figures and tabulation has been stopped at N = 200,000.

As mentioned above the tables were designed as master tables from which optimum plans may be derived for other values of the parameters and for this reason it was decided to tabulate the complete solution with respect to N to make interpolation superfluous.

The user of the tables in practice may easily derive a simplified set of tables from the given ones, either by using a set of fixed N-intervals, or a set of fixed N-arguments. An example has been given in the following table.

Single Sampling Tables for $100p_r = 100p_s = 1.0$, $100p_1 = 0.5$, and $w_2 = 0.05$.

$100p_2$	1.5		2.0		3.0		4.0		5.0		6.0		7.0	
N	n	c	n	c	n	c	n	c	n	c	n	c	n	c
20											Accept		5	0
30									Accept		5	0	5	0
50									5	0	10	0	10	0
70									5	0	10	0	15	0
100							Accept		10	0	15	0	15	0
200							10	0	20	0	20	0	25	0
300							15	0	50	1	50	1	50	1
500					Accept		55	1	60	1	60	1	55	1
700					45	1	65	1	65	1	65	1	60	1
1000					55	1	110	2	105	2	100	2	90	2
2000					125	2	170	3	155	3	140	3	100	2
3000			Accept		195	3	180	3	160	3	145	3	135	3
5000			185	3	265	4	235	4	210	4	185	4	140	3
7000			275	4	330	5	290	5	215	4	190	4	170	4
10000			450	6	400	6	295	5	260	5	195	4	175	4
20000	Accept		640	8	475	7	355	6	310	6	240	5	215	5
30000	755	9	735	9	545	8	410	7	315	6	280	6	220	5
50000	1080	12	920	11	620	9	470	8	365	7	285	6	255	6
70000	1300	14	1095	13	690	10	475	8	410	8	325	7	260	6
100000	1520	16	1190	14	760	11	530	9	415	8	330	7	290	7

The 'natural' parameters of the model are (p_1, p_2, w_2), which characterize the prior distribution, and (p_r, p_s), which depend on the costs. The tables and the properties of the solution will be discussed in terms of these parameters on basis of the results in the next section. However, one property may be stated immediately from the observation that the solution depends on four parameters only, viz. $(p_1, p_2, \gamma_1, \gamma_2)$. The three parameters (p_r, p_s, w_2) may therefore in respect to the solution be considered as functionally related, i.e. <u>combinations of</u> (p_r, p_s, w_2) <u>giving the same</u> (γ_1, γ_2) <u>will lead to the same sampling plan</u>.

From

$$\frac{\gamma_2}{\gamma_1} = \frac{(p_2 - p_r)w_2}{(p_r - p_1)w_1}$$

we find

$$p_r - p_1 = (p_2 - p_1)/(1 + \frac{\gamma_2 w_1}{\gamma_1 w_2}). \qquad (44)$$

From

$$\gamma_1(p_s - p_m) = (p_r - p_1)w_1$$

and

$$p_s - p_m = p_s - p_1 - w_2(p_r - p_1)$$

we find

$$p_s - p_1 = (p_r - p_1)(\frac{w_1}{\gamma_1} + w_2). \qquad (45)$$

These formulas show how p_r and p_s depend on w_2 for given $(p_1, p_2, \gamma_1, \gamma_2)$. To use them in connection with the master tables we put $p_r = p_s$ and $w_2 = 0.05\lambda$ which leads to

$$p_r(\lambda) = p_{1o} + (p_{2o} - p_{1o}) / (1 - \gamma_{2o} + \frac{20\gamma_{2o}}{\lambda})$$

where

$$\gamma_{2o} = \frac{p_{2o} - p_{ro}}{19(p_{ro} - p_{1o})} = \frac{\rho_2 - 1}{19(1 - \rho_1)} ,$$

the index o denoting an argument in the master table, $p_{ro} = 0.01$ or 0.10, $\rho_i = p_{io}/p_{ro}$. Dividing by p_{ro} gives

$$p_r(\lambda)/p_{ro} = \rho_1 + (\rho_2 - \rho_1) / (1 - \gamma_{2o} + \frac{20\gamma_{2o}}{\lambda}) = f(w_2, \rho_1, \rho_2) \tag{46}$$

which has been tabulated in the appendix.

The field of application of the master tables may therefore be considerably enlarged by making use of the following rule:

<u>The optimum sampling plan for</u> $(N, p_{ro}, p_{1o}, p_{2o}, w_2 = 0.05)$, $p_{ro} = p_{so}$, <u>is the same as the plan for</u> $(N, p_{ro} f(w_2, \rho_1, \rho_2), p_{1o}, p_{2o}, w_2)$.

Consider for example the case with $p_{ro} = p_{so} = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, and $w_2 = 0.05$ for which the optimum plans have been given in the master table. The same plans are also optimum for $w_2 = 0.20$, say, and $p_r = p_s = 0.019$, $p_1 = 0.006$, and $p_2 = 0.040$ which may be seen by interpolation in the table of $f(w_2, \rho_1, \rho_2)$ for $\rho_1 = 0.6$ and $\rho_2 = 4.0$.

4. The asymptotic solution.

In this section we shall give a somewhat simpler and more direct proof of the asymptotic results found by Guthrie and Johns [3] and by Hald [4], and furthermore carry the asymptotic expansion so far that we get a useful approximation to the exact solution also for small values of c.

The proof is based on the following lemma which is a special case of a theorem proved by Blackwell and Hodges [5]:

For $c/n = h = p_o + \varepsilon$, p_o being a constant and $\varepsilon \to 0$ for $n \to \infty$, we have

$$P(p) = \frac{1}{\sqrt{2\pi n p_o q_o}} \; \frac{q_o p}{p - p_o} \; e^{-n\varphi(h,p)} (1 + O(\sqrt{\varepsilon})) \text{ for } p_o < p, \tag{47}$$

where

$$\varphi(h,p) = h \ln \frac{h}{p} + (1-h)\ln \frac{1-h}{1-p} . \tag{48}$$

For $p_o > p$ the same expression is valid for $Q(p)$ if only $p - p_o$ is replaced by $p_o - p$.

Writing

$$\varphi(h,p) = \varphi(p_o,p) + \varepsilon\varphi'(p_o,p) + O(\varepsilon^2) \tag{49}$$

where

$$\varphi'(p_o,p) = \ln\frac{p_o q}{q_o p} \tag{50}$$

we find from (26) and (47) the asymptotic expression

$$R = n + (N-n)\frac{q_o}{\sqrt{2\pi n p_o q_o}} \sum_{i=1}^{2} \frac{\gamma_i p_i}{|p_o - p_i|} e^{-n\varphi(p_o,p_i) - n\varepsilon\varphi'(p_o,p_i)}(1 + O(\sqrt{\varepsilon})) \tag{51}$$

on the assumption that $p_1 < p_o < p_2$. (As will be shown later $\varepsilon = O(1/n)$, and we may therefore disregard $n\varepsilon^2$). We shall first determine the value of $h = p_o + \varepsilon$ which minimize R for given n and next determine the value of n giving the absolute minimum by treating R as a differentiable function of h and n.

The essential feature of (51) is that the two binomial risks, $Q(p_1)$ and $P(p_2)$, have been expressed as functions tending exponentially to zero for $n \to \infty$.

As explained in [4] the optimum plan must have the property that $R/N \to 0$ for $N \to \infty$, $n \to \infty$, and $n/N \to 0$. It follows that p_o must satisfy the inequality $p_1 < p_o < p_2$ because otherwise R/N would not tend to zero but to γ_1 or γ_2.

We shall state the theorem to be proved for the double binomial distribution only, but it is valid for a more general class of distributions, viz. for a distribution having probability density $w(p) = 0$ for $p_1 < p < p_2$, $w(p_1) = w_1 > 0$, $w(p_2) = w_2 > 0$, $w_1 + w_2 \leq 1$, and

$$\int_0^{p_1^*} dW(p) + \int_{p_2^*}^{1} dW(p) = 1 - w_1 - w_2$$

for $0 \leq p_1^* < p_1$ and $p_2 < p_2^* \leq 1$, which means that the probability distribution may be arbitrary outside the interval $p_1^* < p < p_2^*$. The result of such a generalization will only be to add a term to (51) of form

$$\frac{N-n}{p_s - p_m} \frac{q_o}{\sqrt{2\pi n p_o q_o}} \int_I \frac{(p_r - p)p}{p_o - p} e^{-n\varphi(h,p)} dW(p),$$

(I denoting the intervals $(0 \leq p \leq p_1^*)$ and $(p_2^* \leq p \leq 1)$) which obviously is $O(e^{-n})$

times the last term of (51) since $\varphi(h,p) > \varphi(h,p_1)$ for $p < p_1$ and $\varphi(h,p) > \varphi(h,p_2)$ for $p > p_2$.

Because of the factor $p_r - p$ in the cost function we might also have assumed that $w(p_r) > 0$ without altering the result.

It is reasonable to assume that the two exponential terms in (51) tend to zero with the same speed, i.e. that p_o is determined from

$$\varphi(p_o,p_1) = \varphi(p_o,p_2)$$

which gives

$$p_o = \ln \frac{q_1}{q_2} \Big/ \ln \frac{p_2 q_1}{q_2 p_1} = \frac{1}{\beta} \tag{52}$$

and

$$\varphi_o = p_o \ln \frac{p_o}{p_i} + q_o \ln \frac{q_o}{q_i}, \quad i = 1 \text{ or } 2. \tag{53}$$

Under this assumption we shall determine ε by minimization of (51). The part of R depending on ε is

$$f(\varepsilon) = \sum_i \frac{\gamma_i p_i}{|p_o - p_i|} e^{-n\varepsilon\varphi'(p_o,p_i)} .$$

From $f'(\varepsilon) = 0$ we find

$$\sum_{i=1}^{2} \frac{\gamma_i p_i \varphi_i'}{|p_o - p_i|} e^{-n\varepsilon\varphi_i'} = 0 \tag{54}$$

where - according to (50) -

$$\varphi_i' = \ln \frac{p_o q_i}{q_o p_i} . \tag{55}$$

Solving for $a = n\varepsilon$ we find

$$a\delta_o' = \ln \frac{\gamma_1 p_1 (p_2 - p_o)\varphi_1'}{\gamma_2 p_2 (p_o - p_1)(-\varphi_2')} \tag{56}$$

where

$$\delta_o' = \varphi_1' - \varphi_2' = \ln \frac{p_2 q_1}{q_2 p_1} . \tag{57}$$

We thus have the result that $c = np_o + a + o(1)$ in accordance with what could be expected from (33).

Inserting these results into (51) we find

$$R = n + (N-n) \frac{\lambda}{\sqrt{n}} e^{-n\varphi_o} \tag{58}$$

with

$$\lambda = \frac{q_o}{\sqrt{2\pi p_o q_o}} \sum_{i=1}^{2} \frac{\gamma_i p_i}{|p_o - p_i|} e^{-a\varphi_i'} . \tag{59}$$

To prove (indirectly) that $h = p_o + \varepsilon$ minimizes R let us assume that $h = p_o + \varepsilon$, given by (52) and (56), does not minimize R but that min R is obtained for $h = h_o + \varepsilon_o$, $h_o \neq p_o$ and $\varepsilon_o \to 0$. Denoting the part of R depending on h by g(h) we find for sufficiently large n and for $h_o < p_o$, say, that

$$g(h_o) = \lambda_1(h_o)\, e^{-n\varphi(h_o,p_1)} (1 + 0(e^{-n}))$$

since $\varphi(h_o,p_2) > \varphi(h_o,p_1)$ for $h_o < p_o$. However, $g(h_o)$ cannot be min g(h) since $\varphi(h_o,p_1) < \varphi(p_o,p_1)$, i.e. we have reached a contradiction by assuming $h_o \neq p_o$.

From dR/dn = 0 we find

$$1 - (N-n)\frac{\lambda}{\sqrt{n}} e^{-n\varphi_o}\left(\varphi_o + \frac{1}{2n}\right) - \frac{\lambda}{\sqrt{n}} e^{-n\varphi_o} = 0 \tag{60}$$

or

$$\ln(N-n) = \varphi_o n + \frac{1}{2}\ln n - \ln(\lambda\varphi_o) + o(1). \tag{61}$$

From (58) and (60) we also have that

$$\min_{(n,c)} R = n + \frac{1}{\varphi_o} + o(1) \tag{62}$$

where n may be determined by inversion of (61), i.e.

$$n = \frac{1}{\varphi_o}\left(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_o\right) + o(1).$$

We have thus found that asymptotically c is a linear function of n and n is proportional to $\ln N - \frac{1}{2}\ln\ln N$ plus a constant. Furthermore it follows from (62) that the average decision loss per lot tends to a constant $1/\varphi_o$ so that for large lots decision losses divided by sampling inspection costs tend to zero.

To investigate the two risks asymptotically we find from (54)

$$\frac{\gamma_1 p_1 \varphi_1'}{p_o - p_1} e^{-a\varphi_1'} = \frac{\gamma_2 p_2(-\varphi_2')}{p_2 - p_o} e^{-a\varphi_2'}$$

so that (59) gives

$$\lambda = \frac{q_o}{\sqrt{2\pi p_o q_o}} \frac{\gamma_1 p_1 \delta_o'}{(p_o - p_1)(-\varphi_2')} e^{-a\varphi_1'}$$

which together with (60) may be used to reduce

$$Q(p_1) = \frac{1}{\sqrt{2\pi p_o q_o}} \frac{q_o p_1}{p_o - p_1} e^{-a\varphi_1'} \frac{1}{\sqrt{n}} e^{-n\varphi_o}$$

$$Q(p_1) = \frac{-\varphi_2'}{\varphi_o \gamma_1 \delta_o'} \; \frac{1}{N-n} \; . \tag{63}$$

Similarly we have

$$P(p_2) = \frac{\varphi_1'}{\varphi_o \gamma_2 \delta_o'} \; \frac{1}{N-n} \tag{64}$$

so that

$$P(p_2)/Q(p_1) = \gamma_1 \varphi_1' / \gamma_2 (-\varphi_2'). \tag{65}$$

We have thus proved the following <u>theorem</u>:

<u>Asymptotically the optimum sampling plan is given by</u>

$$c = np_o + a + o(1) \tag{66}$$

<u>and</u>

$$n = \frac{1}{\varphi_o}(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_o) + o(1) \tag{67}$$

<u>which lead to</u>

$$\min R = \frac{1}{\varphi_o}(\ln N - \frac{1}{2}\ln\ln N + \ln\lambda + \frac{3}{2}\ln\varphi_o + 1) + o(1), \tag{68}$$

$$Q(p_1) = \frac{-\varphi_2'}{\varphi_o \gamma_1 \delta_o'} \; \frac{1}{N-n} + o(\tfrac{1}{N})$$

<u>and</u>

$$P(p_2) = \frac{\varphi_1'}{\varphi_o \gamma_2 \delta_o'} \; \frac{1}{N-n} + o(\tfrac{1}{N}).$$

It will be noted that p_o and φ_o depend on (p_1, p_2) only, i.e. they are independent of the cost parameters and of w_2.

The asymptotic solution supplements the exact one in several respects. Since the optimum plan is a function of 5 parameters $(N, p_1, p_2, \gamma_1, \gamma_2)$ a complete tabulation is rather hopeless even if a program has been worked out for an electronic computer. Furthermore the properties of the exact solution are not easily to be found from the procedure by which the solution is obtained. The advantages of the asymptotic solution are that

(1) it clearly shows how the optimum plan and various derived quantities depend on the parameters,

(2) it may be used as starting point for developing approximations which are valid also for small N,

(3) it may be used for developing interpolation and extrapolation formulas in connection with "master tables" of the exact solution, and

(4) it shows the sensitivity of the solution with respect to changes of the parameters.

These aspects of the solution will be discussed in the following sections.

5. Comparison of exact and approximate solution.

Looking at the relation between n and c in the tables it will be seen that the optimum values of n for a given value of c tend to cluster around

$$n_c = \alpha + \beta\,(c + \tfrac{1}{2}) \tag{69}$$

as might be expected from (33). Comparing with the asymptotic result $c = np_o + a$, $p_o = 1/\beta$ and $\underline{a}$ being defined by (56), agreement between the two expressions would require that

$$\left(\ln \frac{p_2(p_o - p_1)(-\varphi_2')}{p_1(p_2 - p_o)\varphi_1'}\right)\Big/\left(\ln \frac{p_2 q_1}{q_2 p_1}\right) = \frac{1}{2}.$$

It can be proved that the ratio on the left hand side above is positive and less that 1. Numerical investigations show that in typical cases in practice the ratio does not deviate much from 1/2. As examples consider the following results:

$100p_1$	$100p_2$	p_2/p_1	Ratio
0.2	4.0	20	0.528
0.2	2.0	10	0.517
0.6	4.0	6.7	0.512
0.6	2.0	3.3	0.505

The ratio depends primarily on p_2/p_1 and practically the same results will be found for values of (p_1, p_2) which are 10 times as large or 1/lo of the values considered. We shall therefore in the following use the simpler expression (69) instead of $c = np_o + a$ as the starting point for finding n from c or reversely.

The asymptotic formulas may be used in two ways

(1) Starting from c we may determine the corresponding N-interval and within that the relation between n and N.

(2) Starting from N we may determine the corresponding n and from n determine c.

The first method is useful for making a systematic tabulation of sampling plans whereas the second is suitable for computing "isolated" plans for a given N.

Starting from an integer value of c we first find n_c from (69) and the corresponding N_c from (61). Similarly we find $N_{c-0.5}$ and $N_{c+0.5}$, being the lower and upper limit for N having c as optimum acceptance number.

In the asymptotic solution we have disregarded the discreteness of c and n. We may, however, afterwards try to take the effect of the discreteness of c into account by investigating the relationship between n and N for given (integer) value of c. From $dR(N,n,c)/dn = 0$ it can be found that n is approximately a linear function of log N with slope $-1/\log q_2$ *). Within the interval $(N_{c-0.5}, N_{c+0.5})$ we may therefore determine n from the approximate formula

$$n = n_c - (\log N - \log N_c)/\log q_2, \quad N_{c-0.5} < N < N_{c+0.5}, \tag{70}$$

which for small p_2 and small intervals may be replaced by

$$n = n_c + (N - N_c)/N_c p_2, \quad N_{c-0.5} < N < N_{c+0.5} \ . \tag{71}$$

It follows that the values of n belong to the interval

$$n_c \pm \beta(\varphi_o + \frac{1}{2n_c})/2p_2 .$$

For applications in practice we give the formula corresponding to (61) with logarithms to base 10, i.e.

$$\log (N_c - n_c) = \varphi n_c + \frac{1}{2} \log n_c + \delta \tag{72}$$

where

$$\varphi = p_o \log \frac{p_o}{p_i} + q_o \log \frac{q_o}{q_i}, \quad i = 1 \text{ or } 2, \tag{73}$$

$$\delta = -\log (\lambda\varphi_o), \tag{74}$$

and

$$\lambda\varphi_o = 10^{\varphi(\alpha+\frac{\beta}{2})} \frac{\varphi}{\log e} \sqrt{\frac{q_o}{2\pi p_o}} \sum_{i=1}^{2} \frac{\gamma_i p_i}{|p_o - p_i|} \left(\frac{q_i}{q_o}\right)^{\alpha+\frac{\beta}{2}} \tag{75}$$

-a having been replaced by $\frac{\alpha}{\beta} + \frac{1}{2}$ in $\lambda\varphi_o$.

In the following we shall make much use of (72) with $N_c - n_c$ replaced by N_c which only means that we disregard terms of order n_c/N_c and less.

The approximation obtained by using (69), (70), and (72) is usually very good even for quite small values of c. Normally the approximate value of c will deviate at most 1 from the correct value. The approximation depends essentially on p_2/p_1, being good for large values of p_2/p_1 and poorer for small values. Two examples for $p_2/p_1 = 6.7$ and 3.3, respectively, will show the results obtained for a typical good and poor case. Table 1 and Fig. 3 show that the approximate and the exact solution are practically identical in the first

*) This result is due to Mrs. K. West Andersen.

case whereas the approximate solution in the second case often will lead to a value of c being 1 too large and a corresponding value of n.

Table 1.

Comparisons of exact and approximate sampling plans computed from (69),(70), and (72).

$p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$.

$\alpha = -26.7$, $\beta = 55.509$, $\varphi = 0.0034156$, $\delta = 1.5499$, $-1/\log q_2 = 56.405$.

c	Approximation			Exact	
	n_c	n	$N_{c\pm0.5}$	n	N
1	57	43-66	269-714	45-65	280-714
2	112	104-120	715-1400	105-120	715-1420
4	223	216-230	2490-4300	220-230	2550-4390
6	334	328-340	7190-12000	330-340	7390-12300
8	445	439-451	19700-32300	440-450	20200-33000
10	556	550-562	52400-85300	550-560	53600-87000
12	667	661-673	137000-200000	665-670	140000-200000

$p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.020$, $w_2 = 0.05$.

$\alpha = -143.0$, $\beta = 85.879$, $\varphi = 0.0009088$, $\delta = 2.0785$, $-1/\log q_2 = 113.97$.

c	Approximation			Exact	
	n_c	n	$N_{c\pm0.5}$	n	N
2	72	44-83	715-1750	-	-
4	243	234-251	2790-3970	245-250	4420-5590
6	415	408-422	5410-7150	415-420	7100-8980
8	587	581-593	9270-11900	585-595	11300-14200
10	759	753-765	15100-19000	755-765	17700-22000
12	931	925-936	23800-29600	930-935	27300-33700
14	1102	1097-1107	36800-45700	1100-1105	41500-51100
16	1274	1269-1279	56500-69700	1270-1280	62800-77200
18	1446	1441-1451	86000-106000	1445-1450	94600-116000
20	1618	1613-1623	130000-159000	1615-1620	142000-173000

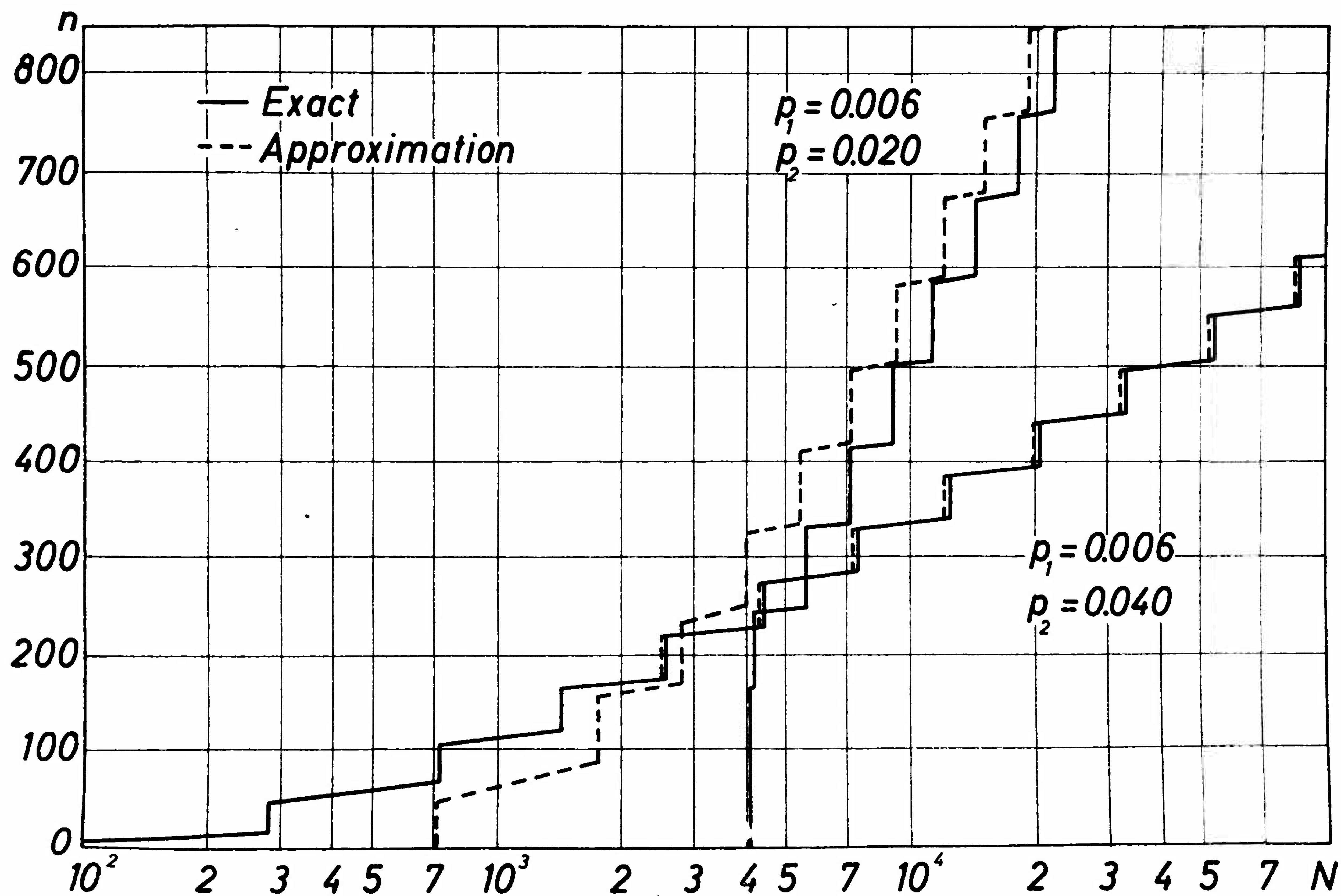

Fig. 3

Comparisons of exact and approximate sampling plans.

It is essential for the efficiency of the approximation to use the right relation between n and c, see the discussion in section 12, and it is therefore fortunate that this relation is a simple linear one.

The approximation formula breaks down for values of N for which the cheapest solution is acceptance without inspection (or rejection without inspection). As will be seen from Table 1 the approximation formula may in such cases lead to a sampling plan even if no optimum plan exists. The difference in costs by using such a plan instead of accepting without sampling inspection will, however, normally be small.

Turning to the inverse formula (67) numerical investigations show that the results are not as accurate as those found from (61). Taking one more term in the inversion of (61) and changing to logarithms with base 10 we find

$$n_N = \frac{1}{\varphi}\{\log N - (\frac{1}{2}\log\log N + d)(1 - \frac{1}{3\log N})\} \tag{76}$$

where

$$d = -\log\lambda\varphi_o - \frac{1}{2}\log\varphi = \delta - \frac{1}{2}\log\varphi. \tag{77}$$

The exact inversion leads to the correction term $(\log e)/(2\log N) = 0.22/\log N$ which, however, on the basis of numerical investigations has been replaced by $1/(3\log N)$. If (76) is to be used extensively it pays to tabulate

$$g(N) = \log N - \frac{1}{2}(1 - \frac{1}{3\log N})\log\log N \tag{78}$$

and use (76) in the form

$$n_N = \frac{1}{\varphi}\{g(N) - d(1 - \frac{1}{3\log N})\}. \tag{79}$$

From n we may then find

$$c_N = p_o(n_N - \alpha) - \frac{1}{2}$$

and round to the nearest integer. To obtain more accurate results n_c may be computed from the rounded value of c_N and n could then be found from (70) or (71).

Table 2 shows that (76) leads to good results for the two previously discussed typical examples.

As a general conclusion of the many numerical comparisons which have been carried out we may state that the asymptotic formulas give sufficiently good approximations to the optimum sampling plans for most practical purposes. If one wants to be sure to find the optimum plan one may start from the approximation and compare the costs of

this plan with the costs of suitably chosen neighbouring plans thus finding the optimum one by trial and error.

Table 2.

Comparisons of exact and approximate sampling plans computed from (76).

$p_r = p_s = 0.010$, $w_2 = 0.05$.

	$p_1 = 0.006$,	$p_2 = 0.040$.			$p_1 = 0.006$,	$p_2 = 0.020$.		
	Approx.		Exact		Approx.		Exact	
N	n_N	c_N	n	c	n_N	c_N	n	c
300	Accept		50	1				
500	20	0	60	1				
700	55	1	65	1				
1000	90	2	115	2				
2000	165	3	170	3				
3000	210	4	220	4	Accept		Accept	
5000	265	5	275	5	180	3	250	4
7000	300	5	285	5	320	5	335	5
10000	345	6	335	6	465	7	505	7
20000	420	8	395	7	755	10	760	10
30000	470	8	450	8	930	12	930	12
50000	525	9	505	9	1145	14	1105	14
70000	565	10	560	10	1290	16	1275	16
100000	610	11	610	11	1445	18	1445	18
200000	690	12	670	12	1750	22	1705	21

The formulas (72) and (77) have, however, the serious drawback from the point of view of application that the constants δ and d are rather hard to compute. The asymptotic formulas have therefore in the following only been used to derive relationships between sampling plans under variation of the parameters. It is to be expected that these relationships will prove to be rather accurate in view of the good approximation demonstrated above.

According to (62) we have for the optimum plans that the average decision loss asymptotically is constant, i.e. $R - n \sim 1/\varphi_o$. For small N this gives an upper limit to the decision loss but the formula is not of much value as an approximation.

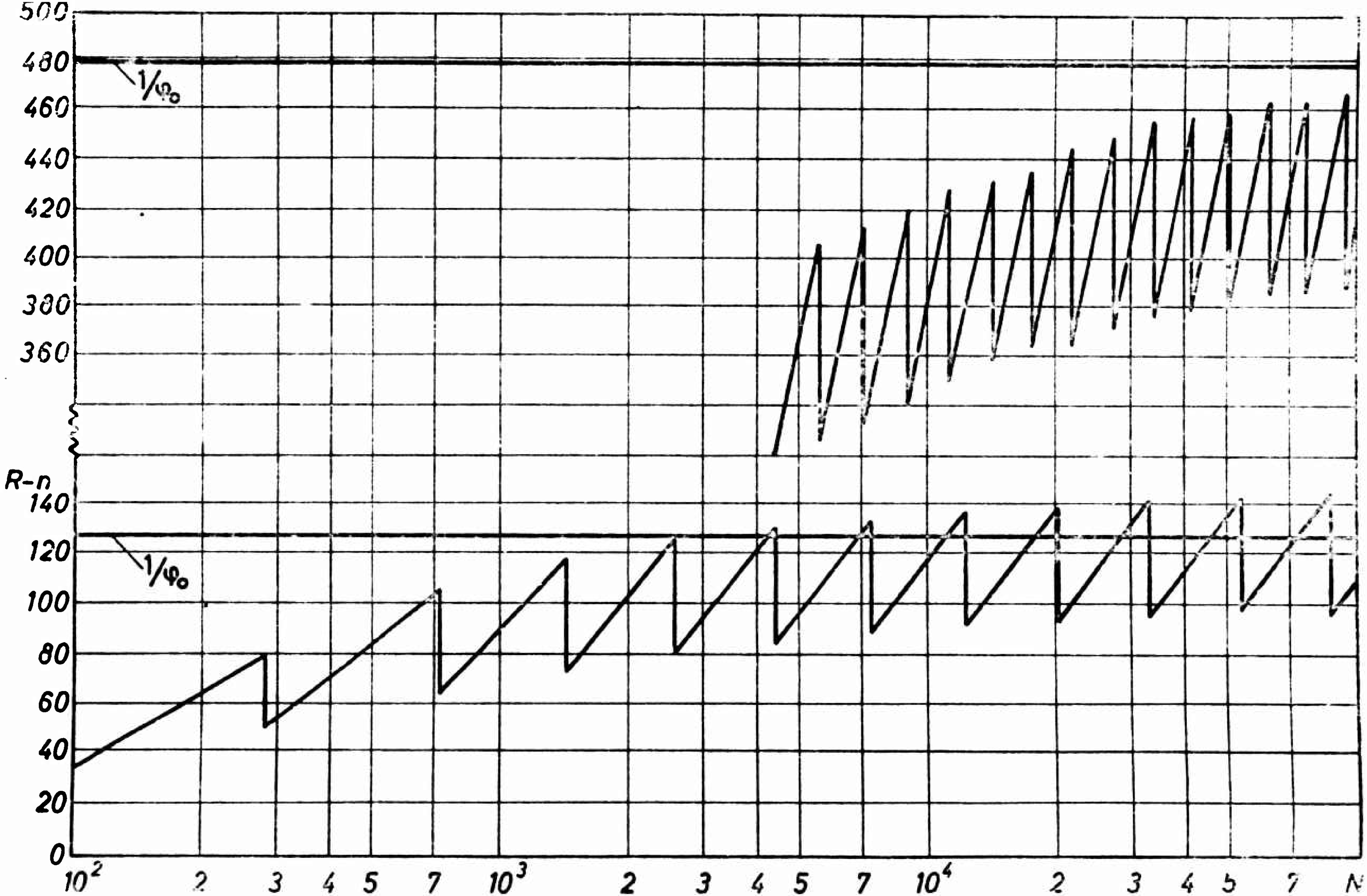

Fig. 4

Average decision loss as function of lot size.

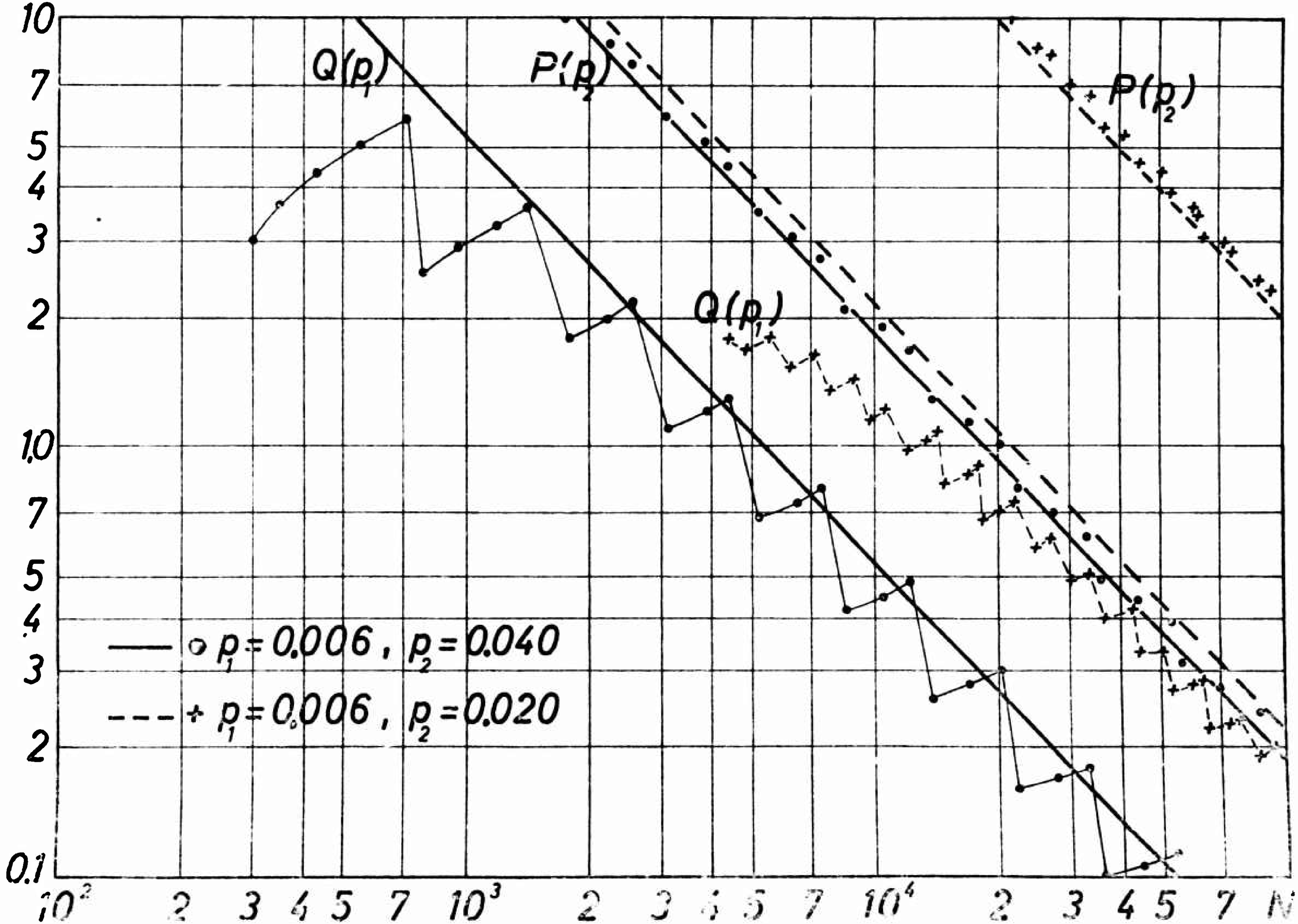

Fig. 5

Probabilities of wrong decisions as functions of lot size.

Fig. 4 sketches for the two previously considered examples R - n as function of N. The discontinuities correspond to changes in c; each time c is increased by 1 n increases approximately by β and R - n decreases with the same quantity. The asymptotic result corresponds to the mid-points of the intervals. It will be seen that the asymptote is nearly being reached for N = 100,000 in the case with $p_2/p_1 = 6.7$ but not for $p_2/p_1 = 3.3$.

For small N a useful upper limit to the average decision loss may be obtained by noticing that $R < N\gamma_2$ if an optimum plan exists and the alternative is acceptance without inspection.

According to (63) and (64) the probabilities of wrong decisions, $Q(p_1)$ and $P(p_2)$, are asymptotically inversely proportional to N. Fig. 5 sketches for the two examples $Q(p_1)$ and $P(p_2)$ as functions of N. The asymptotic formula gives a reasonable approximation to $P(p_2)$ in both cases, whereas the approximation to $Q(p_1)$ is rather poor, particularly for the case $p_2/p_1 = 3.3$. The discontinuities resulting from changes of c are very pronounced for $Q(p_1)$.

6. Proportional change of (p_r, p_s, p_1, p_2) for fixed w_2.

We shall first study the asymptotic formulas for all "quality levels" tending to zero with the same speed. Introducing the auxiliary quantities

$$\rho_s = \frac{p_s}{p_r}, \quad \rho_1 = \frac{p_1}{p_r}, \quad \rho_2 = \frac{p_2}{p_r}, \quad \rho_m = \frac{p_m}{p_r}, \quad \rho = \frac{p_2}{p_1}, \tag{80}$$

we find for $p_r \longrightarrow 0$ and fixed $(\rho_s, \rho_1, \rho_2, w_2)$

$$\alpha p_r \longrightarrow \left(\ln \frac{w_2(\rho_2-1)}{w_1(1-\rho_1)}\right) \Big/ (\rho_2 - \rho_1) = \alpha_o,$$

$$\beta p_r \longrightarrow \left(\ln \frac{\rho_2}{\rho_1}\right) \Big/ (\rho_2 - \rho_1) = \beta_o,$$

$$p_o/p_r \longrightarrow 1/\beta_o = \rho_o,$$

$$\varphi_o/p_r \longrightarrow \rho_o \ln \frac{\rho_o}{\rho_i} + (\rho_i - \rho_o) = \varphi^*, \quad i = 1 \text{ or } 2,$$

and

$$\lambda\varphi_o/\sqrt{p_r} \longrightarrow \exp\{\varphi^*(\alpha_o + \frac{\beta_o}{2})\} \frac{\varphi^*}{\sqrt{2\pi\rho_o}} \sum_{i=1}^{2} \frac{w_i\rho_i(\rho_i-1)}{(\rho_s-\rho_m)(\rho_i-\rho_o)} \exp\{(\rho_o-\rho_i)(\alpha_o+\frac{\beta_o}{2})\}$$

$$= \exp\{-\delta_o\},$$

where in the last expression -a has been replaced by $\frac{\alpha}{\beta} + \frac{1}{2}$ as in (75).

Inserting these results into (69) and (72) we find

$$n_c p_r \longrightarrow \alpha_c + \beta_o (c + \tfrac{1}{2}) = n_o(c)$$

and

$$\ln(N_c p_r) \longrightarrow \varphi^* n_o(c) + \tfrac{1}{2} \ln n_o(c) + \delta_o = \ln N_o(c).$$

It follows that for small p_r we have approximately

$$n_c \sim n_o(c)/p_r$$

and

$$N_c \sim N_o(c)/p_r$$

where $n_o(c)$ and $N_o(c)$ are independent of p_r, i.e. n and N vary inversely proportional to p_r for given c.

Suppose that the optimum sampling plans have been tabulated for a small value of p_r, $p_r = 0.01$ say, and certain values of $(\rho_s, \rho_1, \rho_2, w_2)$. The above result may then be used to find the optimum plans for λp_r, say, from the plans in the given table. Denoting the quantities required by $n_c(\lambda p_r)$ and $N_c(\lambda p_r)$ we have for given c

$$n_c(\lambda p_r) \sim n_c(p_r)/\lambda \qquad (81)$$

and

$$N_c(\lambda p_r) \sim N_c(p_r)/\lambda, \qquad (82)$$

i.e. we have found the following important "proportionality law":

The optimum sampling plan corresponding to $(N, \lambda p_r, \lambda p_s, \lambda p_1, \lambda p_2, w_2)$ is approximately equal to $(n^*/\lambda, c^*)$ where (n^*, c^*) is the plan corresponding to $(N^*, p_r, p_s, p_1, p_2, w_2)$ with $N^* = N\lambda$.

The theorem has been illustrated in Fig. 6 which shows that the approximation holds good also for quite large values of p_r.

This theorem greatly enlarges the field of application of the two master tables. The table with $p_r = 0.01$ may be used for $\lambda < 5$ and the table with $p_r = 0.10$ for $0.5 < \lambda < 2$, in that way covering all cases with $p_r < 0.20$ which is the domain of practical interest.

A large number of numerical investigations has shown that the proportionality law gives rather accurate results. The value of c found will seldom deviate more than 1 from the correct value. For $\lambda > 1$ the formula will normally tend to give too large a value of c and for $\lambda < 1$ too small a value.

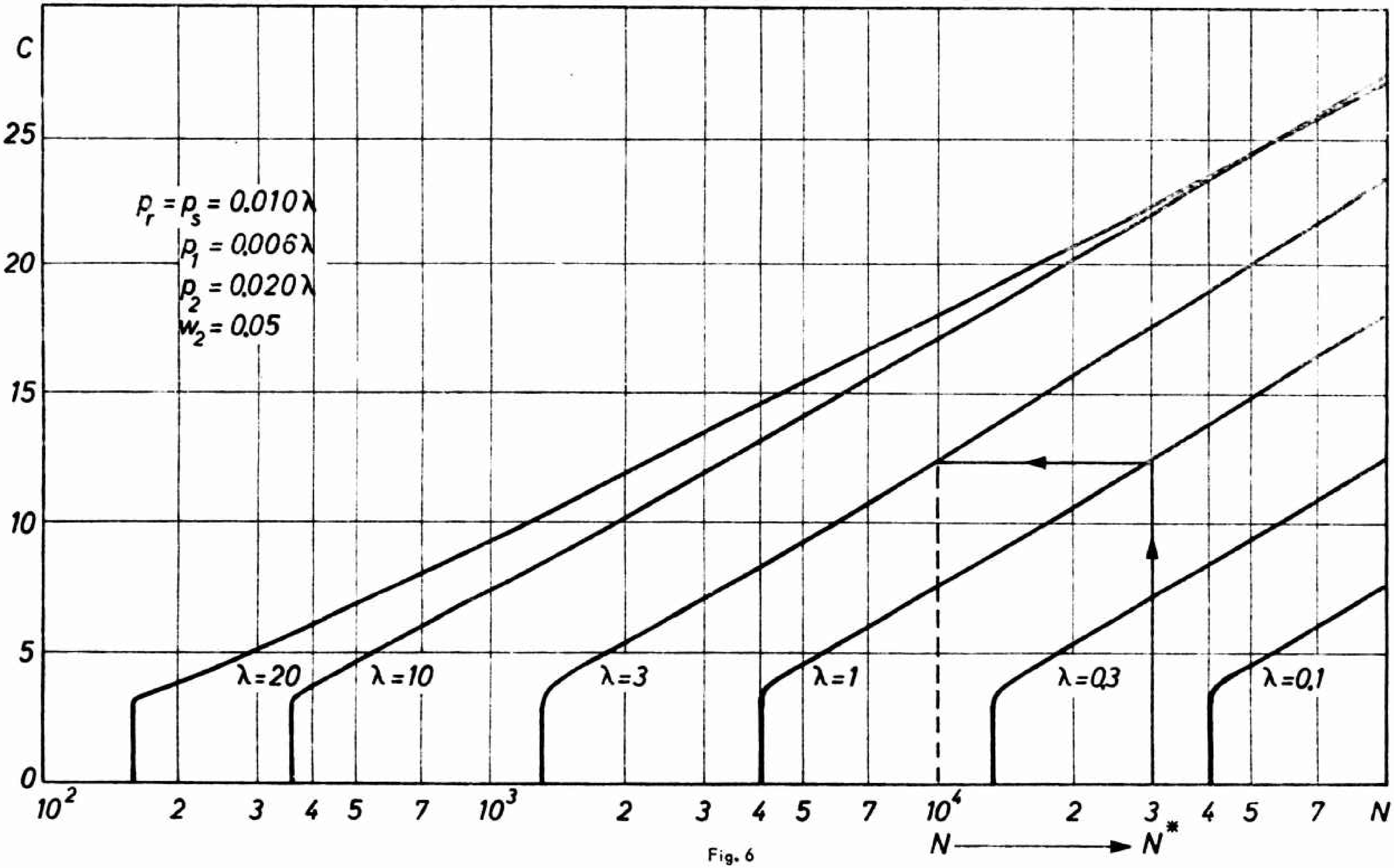

Fig. 6

Relation between lot size and acceptance number by proportional change of (P_r, P_s, P_1, P_2) for fixed w_2.

Table 3.

Comparisons of exact sampling plans for $p_r = p_s = 0.030$, $p_1 = 0.018$ $p_2 = 0.060$, $w_2 = 0.05$, and approximate plans derived from the master tables by the proportionality law.

	Exact		Derived from $p_r = 0.01$ ($\lambda = 3$)			Derived from $p_r = 0.10$ ($\lambda = 0.3$)		
N	n	c	N*=3N	n*/3	c*	N*=0.3N	n*/0.3	c*
1000	Accept		3000	Accept		300	Accept	
2000	110	5	6000	110	5	600	115	5
3000	140	6	9000	165	7	900	145	6
5000	225	9	15000	225	9	1500	200	8
7000	255	10	21000	255	10	2100	255	10
10000	310	12	30000	310	12	3000	285	11
20000	395	15	60000	395	15	6000	365	14
30000	455	17	90000	455	17	9000	425	16
50000	510	19	150000	540	20	15000	480	18
70000	570	21	210000	570	21	21000	535	20
100000	625	23	-	-	-	30000	565	21
200000	710	26	-	-	-	60000	675	25

The example in Table 3 shows the derivation of sampling plans with a break-even quality of $p_r = 0.03$, partly from the first master table using $\lambda = 3$ and partly from the second using $\lambda = 0.3$. Both results are remarkably close to the exact solution, see also Fig. 6.

Consider now the inverse formula (79).

From

$$d + \log p_r \longrightarrow \delta_o \log e - \frac{1}{2} \log \varphi^* = d_o$$

we find

$$n_N(\lambda p_r) \sim \frac{n_N(p_r)}{\lambda} + \left(1 - \frac{1}{3 \log N}\right) \frac{\log \lambda}{\lambda \varphi(p_r)} \qquad (83)$$

where $\varphi(p_r)$ denotes the value of φ for the given (basic) set of parameters. This formula shows how the sample size for a given lot size changes with the "quality level". This result is, however, not as accurate as the previous one for small N and it is neither as convenient for use in connection with the tables.

An example has been given in the following table for N = 50,000, $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, and $w_2 = 0.05$.

Comparisons of exact and approximate sampling plans derived from (83).

	Exact		Approximation	
λ	n	c	n	c
0.1	1850	3	2330	4
0.3	1300	7	1210	7
1.0	505	9	-	-
3.0	205	11	210	11
10.0	63	12	78	15

7. Change of p_s for fixed (p_r, p_1, p_2, w_2).

The master tables contain sampling plans for $p_s = p_r$ only, because a simple and rather accurate rule exists for deriving plans for $p_s \neq p_r$ from the tabulated ones.

From (69) and (72) it will be seen that p_s influences N_c only through δ. Writing

$$p_s - p_m = p_s - p_r + w_1(p_r - p_1)$$

$$= w_1(p_r - p_1)\left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)$$

it follows from (72) that

$$\log N_c(p_r, p_s) = \log N_c(p_r, p_r) + \log\left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)$$

or

$$N_c(p_r, p_s) = N_c(p_r, p_r)/\lambda_s, \tag{84}$$

say, where

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)^{-1} . \tag{85}$$

We have thus proved the following theorem:

The optimum sampling plan corresponding to $(N, p_r, p_s, p_1, p_2, w_2)$ is approximately equal to the plan (n^*, c^*) corresponding to $(N^*, p_r, p_r, p_1, p_2, w_2)$ with $N^* = N\lambda_s$.

This theorem makes it possible to use the master tables also for $p_s \neq p_r$ if only N is replaced by N^*. The error in c by using this procedure will seldom be more than $\pm$ 1. An example has been given in Table 4 with

$$\lambda_s = \left(1 + \frac{2 - 1}{0.95(1 - 0.6)}\right)^{-1} = 0.275.$$

Table 4.

Comparisons of exact sampling plans for $p_r = 0.010$, $p_s = 0.020$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$ with approximate plans derived from the master table.

	Exact		Approximation		
N	n	c	$N^* = 0.275N$	n^*	c^*
300	Accept		83	5	0
500	5	0	138	10	0
700	10	0	193	15	0
1000	15	0	275	15	0
2000	60	1	550	60	1
3000	110	2	825	110	2
5000	120	2	1380	120	2
7000	170	3	1930	170	3
10000	220	4	2750	220	4
20000	280	5	5500	280	5
30000	330	6	8250	330	6
50000	390	7	13800	390	7
70000	395	7	19300	395	7
100000	450	8	27500	445	8
200000	505	9	55000	550	10

The corresponding "inverse" formula becomes

$$n_N(p_r,p_s) = n_N(p_r,p_r) + \frac{1}{\varphi}\left(1 - \frac{1}{3\log N}\right)\log \lambda_s . \tag{86}$$

Using this result for N = 50000 and the parameters given in Table 4 we find

$$n = 505 - 293 \times 0.9291 \times 0.5607 = 350$$

as compared to the exact solution 390.

In the following sections we shall limit ourselves to consider cases with $p_s = p_r$ since we may always begin the analysis by replacing N by N^* if $p_s \neq p_r$. The "conversion factor" λ_s depends on w_2 and the ratios (ρ_s,ρ_1), i.e. λ_s is independent of p_2 and the general quality level.

8. Proportional change of (p_r,p_1,p_2) and change of w_2.

Consider the problem of finding the optimum plans for an arbitrary set of parameter values (p_r,p_1,p_2,w_2) by combining the proportionality law with the relation between p_r and w_2 for given γ_2 and using the tabulated plans in the master table for parameter values $(p_{ro},p_{1o},p_{2o},w_{2o})$, say.

The problem is to determine λ so that $(p_r,p_1,p_2) = (\lambda p^*_{ro}, \lambda p_{1o}, \lambda p_{2o})$ and $(p^*_{ro}, p_{1o},p_{2o},w_2)$ give the same value of γ_2 as $(p_{ro},p_{1o},p_{2o},w_{2o})$. For this value of λ we may find the plans for (p_r,p_1,p_2,w_2) from the plans for $(p^*_{ro}, p_{1o},p_{2o},w_2)$ by means of the proportionality law, and the plans for $(p^*_{ro}, p_{1o},p_{2o},w_2)$ are identical to the plans for $(p_{ro},p_{1o},p_{2o},w_{2o})$. (It will be noted that p^*_{ro}/p_{ro} is identical to the function defined by (46)).

Since the value of γ_2 is the same for (p_r,p_1,p_2,w_2) and $(p^*_{ro},p_{1o},p_{2o},w_2)$ we have the equation $\gamma_{2o} = \gamma_2$ for the determination of λ, i.e.

$$\frac{w_{2o}(p_{2o}-p_{ro})}{w_{1o}(p_{ro}-p_{1o})} = \frac{w_2(p_2-p_r)}{w_1(p_r-p_1)} .$$

Introducing $p_{2o} = p_2/\lambda$ and $p_{1o} = p_1/\lambda$ we find

$$\lambda p_{ro} = \left(p_2 + \frac{w_{1o}}{w_{2o}}\gamma_2 p_1\right) \Big/ \left(1 + \frac{w_{1o}}{w_{2o}}\gamma_2\right) . \tag{87}$$

For the master table with $p_{ro} = 0.01$ and $w_{2o} = 0.05$ the result is

$$\lambda = 100(p_2 + 19\gamma_2 p_1)/(1 + 19\gamma_2) . \tag{88}$$

For the other master table (p_{ro} = 0.10) the factor 100 should be replaced by 10.

The results of sections 6-8 may be combined to the following theorem:

The optimum sampling plan corresponding to (N, p_r, p_s, p_1, p_2, w_2) is approximately equal to (n^*/λ, c^*) where (n^*, c^*) may be found in the master table for $N^* = N\lambda_s\lambda$, $p_{1o} = p_1/\lambda$, and $p_{2o} = p_2/\lambda$, the conversion factors being equal to

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)^{-1}$$

and

$$\lambda = 100(p_2 + 19\,\gamma_2\, p_1)/(1 + 19\,\gamma_2), \qquad \gamma_2 = \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)},$$

for the 0.01-table, 100 being replaced by 10 for the 0.10-table.

By means of this theorem it is rather easy to find the optimum plan corresponding to an arbitrary set of parameter values if only p_1/λ and p_2/λ fall within the range of arguments in the master tables. If that is not the case the method given in the next section may be used.

Usually p_1/λ and p_2/λ will not be equal to the arguments used in the master tables. One might then interpolate but this is hardly worth while since the arguments in the table have been chosen in such a way that by rounding to the nearest argument the rounding error will ordinarily be less than 10 %.

If one wants to be sure to obtain a sufficiently large sample the value of p_1/λ should be rounded up and the value of p_2/λ rounded down.

As an example consider the problem of finding the sampling plans for (p_r,p_1,p_2,w_2) = (0.03, 0.01, 0.07, 0.08) and $p_s = p_r$. Since $p_r < 0.05$ say, we choose to use the the 0.01-table. From

$$\gamma_2 = \frac{8}{92}\,\frac{7-3}{3-1} = 0.174, \qquad 19\,\gamma_2 = 3.31,$$

we find $\lambda = (7 + 3.31)/(1 + 3.31) = 2.39$, $p_1/\lambda = 0.01/2.39 = 0.0042$, and $p_2/\lambda = 0.07/2.39 = 0.029$. The master table should thus be entered with $p_{1o} = 0.004$ and $p_{2o} = 0.030$. For $N = 2000$, say, we find $N^* = 4780$ and $(n^*,c^*) = (210, 3)$ leading to $(n,c) = (210/2.39, 3) = (90,3)$ which is the correct solution.

If $w_2 = 0.02$ instead of 0.08 we find similarly $\lambda = 4.38$, $p_1/\lambda = 0.0023 \cong 0.0025$ and $p_2/\lambda = 0.0160 \cong 0.0150$. For $N = 2000$ we get $N^* = 8760$ leading to acceptance without inspection as the most economical decision.

<u>9. Change of</u> w_2 <u>for fixed</u> (p_r, p_s, p_1, p_2).

In the following we shall develop a method for evaluating the effect of changing one of the five parameters only, and use it first for w_2 and then for p_r.

From (69) and (72) we find for given c

$$\frac{\partial n_c}{\partial \log w_2} = \frac{\partial \alpha}{\partial \log w_2} = 1/(w_1 \log \frac{q_1}{q_2}) \tag{89}$$

and

$$\frac{\partial \log N_c}{\partial \log w_2} = \varphi \frac{\partial n_c}{\partial \log w_2} + \frac{1}{2} \frac{\partial \log n_c}{\partial \log w_2} + \frac{\partial \delta}{\partial \log w_2}. \tag{90}$$

The last term om the right hand side is a rather complicated function of the parameters. Tabulation of δ and graphical analysis of δ as a function of $\log w_2$ has shown, however, that at least for $w_2 \leq 0.20$ and $p_r \leq 0.10$ (and corresponding values of p_s, p_1, p_2) <u>δ is approximately a linear function of</u> $\log w_2$ <u>with a slope depending slightly on</u> (ρ_s, ρ_1, ρ_2) <u>and being practically independent of</u> p_r.

Limiting ourselves to the case $p_s = p_r$ we thus have

$$\frac{\partial \delta}{\partial \log w_2} \cong - b_1(\rho_1, \rho_2),$$

say, where $b_1(\rho_1, \rho_2)$ has been tabulated in the appendix.

Writing $\delta = \delta(p_r, \rho_1, \rho_2, w_2)$ and putting $w_2 = 0.02$ and 0.20 respectively, so that so that $\Delta \log w_2 = \log 0.20 - \log 0.02 = 1$, an approximation to $\partial\delta/\partial \log w_2$ may be found as $\delta(p_r, \rho_1, \rho_2, 0.20) - \delta(p_r, \rho_1, \rho_2, 0.02)$. This approximation has been computed for both $p_r = 0.01$ and 0.10, and finally the average of the two has been taken as $-b_1$.

For small p_r we also have

$$\varphi / \log \frac{q_1}{q_2} \cong \varphi^* / (\rho_2 - \rho_1) = b_2(\rho).$$

The values given for b_2 have been computed as averages of $\varphi/(\log \frac{q_1}{q_2})$ for $p_r = 0.01$ and $p_r = 0.10$.

For large n we have that $(\log c)/2n$ is small as compared to φ and we shall therefore disregard the second term on the right hand side of (90). We then have approximately

$$\frac{\partial \log N_c}{\partial \log w_2} = \frac{b_2(\rho)}{w_1} - b_1(\rho_1, \rho_2)$$

which gives

$$N_c(w_2) = Aw_2^{-b_1}\left(\frac{w_1}{w_2}\right)^{-b_2}$$

where A denotes a constant of integration. Changing from w_2 to λw_2 we get

$$N_c(\lambda w_2) = N_c(w_2)/f_1(\lambda) \tag{91}$$

where

$$f_1(\lambda) = \lambda^{b_1-b_2}\left(1 - (\lambda - 1)\frac{w_2}{w_1}\right)^{b_2} . \tag{92}$$

From (69) we further have

$$n_c(\lambda w_2) = n_c(w_2) + g_1(\lambda) \tag{93}$$

where

$$g_1(\lambda) = \left(\log \frac{\lambda w_1}{1-\lambda w_2}\right)\bigg/\left(\log \frac{q_1}{q_2}\right) \cong \left(\ln \frac{\lambda w_1}{1-\lambda w_2}\right)\bigg/(\rho_2-\rho_1)p_r . \tag{94}$$

For convenience f_1 and g_1 have been written as functions of λ only, even if they both depend also on other parameters. The function f_1 which will be called <u>the conversion factor for N due to a change in</u> w_2 has been tabulated in the appendix for $w_2 = 0.05$ as a function of $(\lambda, \rho_1, \rho_2)$. The function g_1 which gives <u>the correction to n due to a change in</u> w_2 has similarly been tabulated in the appendix as function of $(\lambda, \rho_1, \rho_2)$ for $w_2 = 0.05$ <u>and</u> $p_r = 0.01$. Values of this function for other values of p_r may be obtained as $g_1/100p_r$ where g_1 represents the tabulated values.

The above results may be formulated as the following <u>theorem:</u>

<u>The optimum sampling plan corresponding to</u> $(N, p_r, p_s, \rho_1, \rho_2, \lambda w_2)$, $p_r = p_s$, <u>is approximately equal to</u> $(n^* + g_1(\lambda), c^*)$ <u>where</u> (n^*, c^*) <u>is the plan corresponding to</u> $(N^*, p_r, p_s, \rho_1, \rho_2, w_2)$ <u>with</u> $N^* = Nf_1(\lambda)$.

The theorem has been illustrated in Fig. 7.

This theorem enlarges the field of application of the two master tables with respect to values of w_2 in a similar manner as the law of proportionality does with respect to the other parameters. The results of using the approximation have been compared with the exact solutions in a large number of cases and the deviations found between the approximate and the correct value of c have never exceeded 1 for $\lambda < 4$. There is a tendency for the approximation to give too small a value of c for $\lambda > 1$ and too large a value for $\lambda < 1$, in particular for small N.

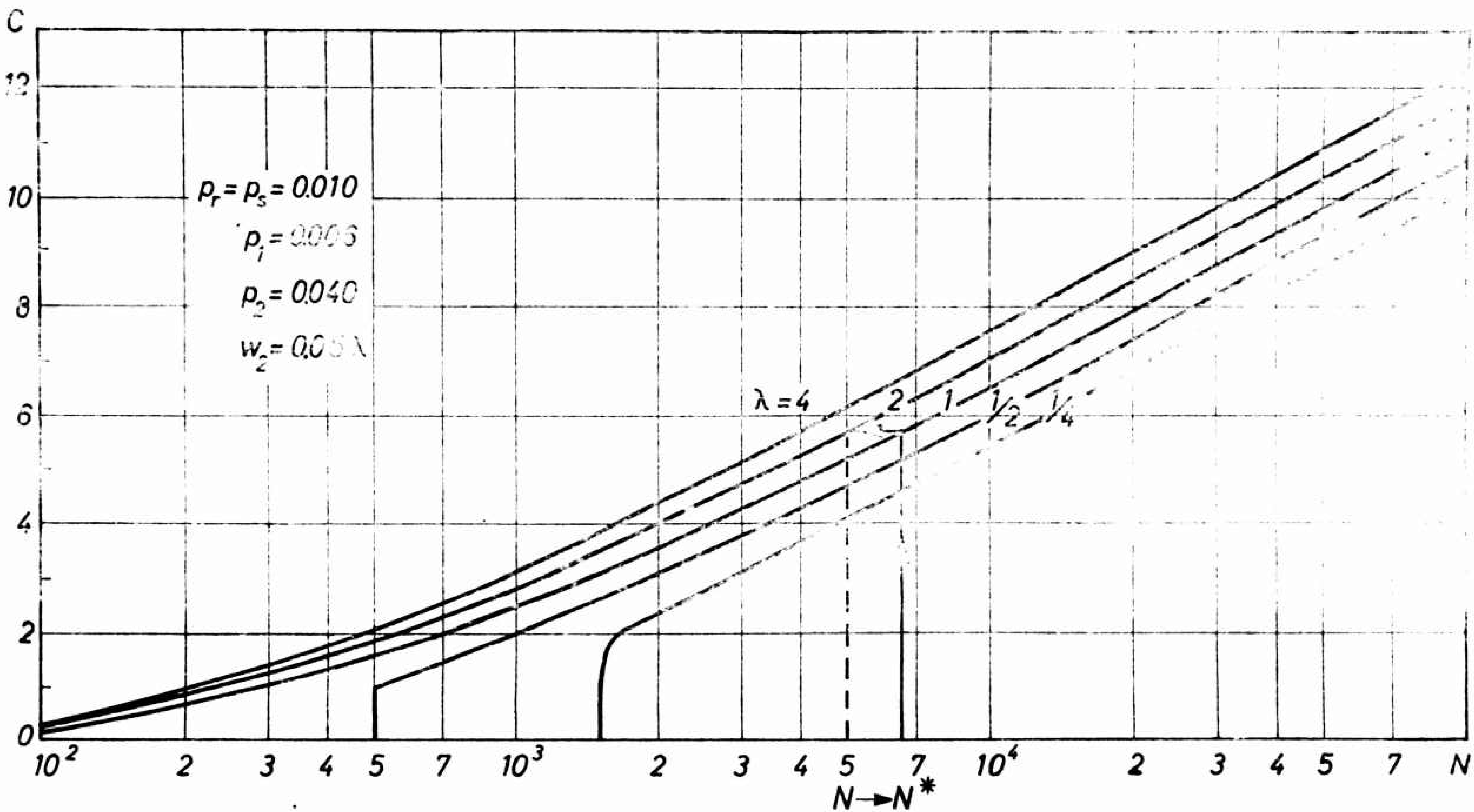

Fig. 7

Relation between lot size and acceptance number by change of w_2 for fixed (P_r, P_s, P_1, P_2)

It should be noted that the formula breaks down in some cases for small N. Let N_a denote the largest N for which acceptance without inspection is cheaper than sampling inspection for the master table used. If $\lambda > 1$ and $Nf_1(\lambda) = N^* < N_a$ then the formula does not lead to a sampling plan even if there may exist a plan which for λw_2 is cheaper than acceptance without inspection. Similarly, for $\lambda < 1$ and $Nf_1(\lambda) = N^* > N_a$ there may be some cases where the approximation formula leads to a sampling plan even if the cheapest solution is acceptance without inspection.

An example has been shown in Table 5. The approximation is remarkably good. Since $N_a = 74$ the approximation formula leads to acceptance without inspection for all $N \leq 57$. Sampling plans cheaper than acceptance without inspection do, however, exist for $12 \leq N \leq 57$.

Using the method of section 8 we find $\gamma_2 = 0.833$, $\lambda = 0.804$, $p_1/\lambda = 0.0075$, and $p_2/\lambda = 0.050$. Since p_1/λ falls outside the range of arguments in the master table the method does not apply. Using $p_1/\lambda = 0.007$ gives, however, a rather good approximation.

Table 5.

Comparisons of exact sampling plans for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.10$, and approximate plans derived from the master table. $f_1 = 1.29$, $g_1 = 20$.

	Exact		Approximate		
N	n	c	N*= 1.29N	n*+ 20	c*
50	15	0	65	Accept	
70	20	0	90	25	0
100	25	0	129	30	0
200	35	0	258	35	0
300	75	1	387	75	1
500	85	1	645	85	1
700	130	2	903	130	2
1000	140	2	1290	140	2
2000	240	4	2580	240	4
3000	250	4	3870	250	4
5000	305	5	6450	300	5
7000	355	6	9030	355	6
10000	405	7	12900	405	7
20000	465	8	25800	465	8
30000	520	9	38700	520	9
50000	575	10	64500	575	10
70000	630	11	90300	630	11
100000	635	11	129000	635	11
200000	740	13	-	-	-

From the inverse formula (79) we get

$$\frac{\partial n_N}{\partial \log w_2} = \frac{b_1(\rho_1,\rho_2)}{\varphi}\left(1 - \frac{1}{3 \log N}\right)$$

and consequently

$$n_N(w_2) = A + \frac{b_1}{\varphi}\left(1 - \frac{1}{3 \log N}\right) \log w_2$$

or

$$n_N(\lambda w_2) = n_N(w_2) + \frac{b_1}{\varphi}\left(1 - \frac{1}{3 \log N}\right) \log \lambda \ . \tag{95}$$

This shows that the difference between $n(\lambda w_2)$ and $n_N(w_2)$ for given N is proportional to $\log \lambda$. This formula is, however, not as accurate as (93) for small N.

An example has been given in the following table for $N = 20{,}000$, $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.020$, and $w_2 = 0.05$, which gives $b_1 = 0.59$ and $1/\varphi = 1100$.

Comparisons of exact and approximate sampling plans derived from (95).

		Exact		Approx.	
$100\ w_2$	λ	n	c	n	c
2.5	0.5	540	8	580	9
5.0	1.0	760	10	-	-
10.0	2.0	980	12	940	11
20.0	4.0	1130	13	1120	13

10. Change of $p_r = p_s$ for fixed (p_1, p_2, w_2).

From (69) and (72) we find for given c

$$\frac{\partial n_c}{\partial \log p_r} = \frac{\partial \alpha}{\partial \log p_r} = -p_r \left(\frac{1}{p_r - p_1} + \frac{1}{p_2 - p_r}\right) \Big/ \left(\log \frac{q_1}{q_2}\right) \tag{96}$$

and

$$\frac{\partial \log N_c}{\partial \log p_r} = \varphi \frac{\partial n_c}{\partial \log p_r} + \frac{1}{2} \frac{\partial \log n_c}{\partial \log p_r} + \frac{\partial \delta}{\partial \log p_r} \ . \tag{97}$$

Numerical investigations show that - for $w_2 < 0.20$, $p_r < 0.20$, and $p_r = p_s$ - δ <u>is approximately a linear function of $\log p_r$ with a slope depending on ρ and being practically independent of "the level of (p_1,p_2)" and of w_2 if only p_r does not come too close to</u> p_1 <u>or</u> p_2, i.e.

$$\frac{\partial \delta}{\partial \log p_r} \simeq b_3(\rho) \quad \text{for} \quad p_1 \rho^{1/5} < p_r < p_2 \rho^{-1/5} \ ,$$

say, where $b_3(\rho)$ has been tabulated. (Another limitation of no practical importance is that p_2 must not be too close to 1). An approximation to $\partial\delta/\partial\log p_r$ may be computed as the corresponding difference - quotient setting $p_r = p_1\rho^{1/5}$ and $p_r = p_2\rho^{-1/5}$ respectively. This has been done for $w_2 = 0.05$ and for the "standard" values of p_1 and p_2, partly at the 1 % and partly at the 10 % level. The value of b_3 given in the table is the average of the two values found.

Proceeding as in section 9 we have approximately

$$\frac{\partial\log N_c}{\partial\log p_r} = \cdot\, b_2(\rho)p_r\left(\frac{1}{p_r-p_1} + \frac{1}{p_2-p_r}\right) + b_3(\rho)$$

which on integration gives

$$N_c(p_r) = Ap_r^{b_3}\cdot\left(\frac{p_2-p_r}{p_r-p_1}\right)^{b_2}$$

and

$$N_c(\lambda p_r) = N_c(p_r)/f_2(\lambda) \tag{98}$$

where

$$f_2(\lambda) = \lambda^{-b_3}\left(\frac{(\rho_2-1)(\lambda-\rho_1)}{(1-\rho_1)(\rho_2-\lambda)}\right)^{b_2} \quad . \tag{99}$$

From (69) we further have

$$n_c(\lambda p_r) = n_c(p_r) + g_2(\lambda) \tag{100}$$

where

$$g_2(\lambda) \approx \left(\ln\frac{(1-\rho_1)(\rho_2-\lambda)}{(\rho_2-1)(\lambda-\rho_1)}\right)\bigg/(\rho_2-\rho_1)p_r. \tag{101}$$

<u>The conversion factor for N due to a change in</u> p_r, $f_2(\lambda)$, has been tabulated in the appendix as function of $(\lambda, \rho_1, \rho_2)$, and <u>the correction to n due to a change in</u> p_r, $g_2(\lambda)$, has been tabulated as function of $(\lambda, \rho_1, \rho_2)$ for $p_r = 0.01$. Values of $g_2(\lambda)$ for other values of p_r may be found from the tabulated ones by dividing by $100p_r$.

The above results may be formulated as the following <u>theorem</u>:

<u>The optimum sampling plan corresponding to</u> $(N, \lambda p_r, \lambda p_s, p_1, p_2, w_2)$, $p_r = p_s$, <u>is approximately equal to</u> $(n^* + g_2(\lambda), c^*)$ where (n^*, c^*) <u>is the plan corresponding to</u> $(N^*, p_r, p_s, p_1, p_2, w_2)$ <u>with</u> $N^* = Nf_2(\lambda)$.

With the given set of tables this theorem is, however, not as important in practice as the previous ones, because the tables contain the optimum plans for so many combinations of (p_r, p_1, p_2) that an adjustment of the relative position of p_r within the interval (p_1, p_2) will seldom be felt necessary from a practical point of view.

In table 6 an example has been shown of the effect of changirg $p_r = p_s$ from 0.010 to 0.020 within the interval $(p_1, p_2) = (0.006, 0.040)$.

Table 6.

Comparisons of exact sampling plans for $p_r = p_s = 0.020$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$ with approximate plans derived from the master table. $\lambda = 2$, $f_2 = 0.52$, $g_2 = -55$.

N	Exact n	Exact c	Approximate $N^* = 0.52N$	Approximate n^*-55	c^*
2000	Accept		1040	60	2
3000	75	2	1560	110	3
5000	130	3	2600	165	4
7000	180	4	3640	170	4
10000	230	5	5200	225	5
20000	290	6	10400	280	6
30000	345	7	15600	335	7
50000	400	8	26000	390	8
70000	450	9	36400	445	9
100000	460	9	52000	450	9
200000	565	11	104000	555	11

From the inverse formula (79) we get

$$\frac{\partial n_N}{\partial \log p_r} = -\frac{b_3(\rho)}{\varphi}\left(1 - \frac{1}{3\log N}\right)$$

which leads to

$$n_N(\lambda p_r) = n_N(p_r) - \frac{b_3}{\varphi}\left(1 - \frac{1}{3\log N}\right)\log\lambda. \qquad (102)$$

An example of the application of this formula has been given in the following table for $N = 50{,}000$, $p_r = p_s = 0.010$, $\lambda = 0.5$ and 2.5, $p_1 = 0.002$, $p_2 = 0.040$, and $w_2 = 0.05$, which give $b_3 = 1.09$ and $1/\varphi = 177$.

Comparisons of exact and approximate sampling plans derived from (102).

p_r	λ	Exact n	Exact c	Approximate n	Approximate c
0.005	0.5	350	4	370	4
0.010	1.0	315	4	-	-
0.025	2.5	250	4	245	4

11. Change of all parameters.

The results of the preceding sections may be combined intc a "chain formula" of the type

$$N_c(\lambda p_r, \rho_s\lambda p_r, \lambda p_1, \lambda p_2, \lambda_1 w_2) = N_c(p_r, p_r, p_1, p_2, w_2)/\lambda_s f_1 \lambda \qquad (103)$$

and

$$n_c(\lambda p_r, \rho_s\lambda p_r, \lambda p_1, \lambda p_2, \lambda_1 w_2) = (n_c(p_r, p_r, p_1, p_2, w_2) + g_1)/\lambda \qquad (104)$$

where

$$\lambda_s = \left(1 + \frac{\rho_s - 1}{(1-\lambda_1 w_2)(1-\rho_1)}\right)^{-1},$$

$f_1(\lambda_1)$ and $g_1(\lambda_1)$ being defined by (92) and (94) for $\rho_1 = p_1/p_r$ and $\rho_2 = p_2/p_r$.

In the master tables $p_r = p_s = 0.01$ (or 0.10) and $w_2 = 0.05$ have been used as reference values. What has been denoted by λ and λ_1 in the above formulas become $100p_r$ (or $10p_r$) and $20w_2$ if p_r and w_2 denote the values for which the optimum plan is required.

We thus get the following rule for using the master table with $p_r = 0.01$:

<u>The optimum plan for</u> $(N, p_r, p_s, p_1, p_2, w_2)$ <u>with</u> $p_r < 0.05$ <u>is approximately equal to</u> $((n^* + g_1)/100p_r, c^*)$ <u>where</u> (n^*, c^*) <u>may be found by entering the master table with</u>

$$N^* = N(100p_r)f_1(20w_2, \rho_1, \rho_2) \Big/ \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right), \qquad (105)$$

$\rho_1 = p_1/p_r$, $\rho_2 = p_2/p_r$, <u>and</u> $g_1 = g_1(20w_2, \rho_1, \rho_2)$, <u>the arguments for</u> (p_1, p_2) <u>in the master table being</u> $(\rho_1/100, \rho_2/100)$.

For $0.05 < p_r < 0.20$ the master table with $p_r = 0.10$ should be used accordingly.

If $(\rho_1/100, \rho_2/100)$ are not to be found in the table then use the "nearest" argument or interpolate. One may also use the results in section 10 to change p_r in the master table so that the relations between (p_r, p_1, p_2) in the table become closer to the ones for which the sampling plan is required. From a practical point of view, however, the master tables combined with the rule above will normally suffice.

An example has been given in Table 7. The conversion factor for N is found as

$$3f_1(2, 0.6, .4.0) \;/\; \left(1 + \frac{30}{0.90 \times 12}\right) = 3 \times 1.29/3.78 = 1.02.$$

The agreement between the approximate and the exact solution is very good.

Using instead the method of section 8 we get $\lambda_s = 1/3.78 = 0.265$, $\lambda = 2.40$, $p_1/\lambda = 0.0075$, and $p_2/\lambda = 0.050$, i.e. $N^* = 0.636N$ and $n = n^*/2.40$. Since the master table does not contain the argument 0.0075 we may as an approximation use 0.0070 which, however, will tend to give too small values of c.

The corresponding inverse formula takes the form

$$n_N(p_r, p_s, p_1, p_2, w_2) = \frac{1}{100p_r}\Big[n_o + \frac{1}{\varphi}\Big\{\log(100p_r) + b_1\log(20w_2)$$
$$- \log\left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)\Big\}\left(1 - \frac{1}{3\log N}\right)\Big] \qquad (106)$$

where n_o denotes the sample size to be found in the master table for $p_r = 0.01$, $\rho_1 = p_1/p_r$, $\rho_2 = p_2/p_r$, corresponding to the given lot size N.

Table 7.
Comparisons of exact sampling plans for $p_r = 0.030$, $p_s = 0.060$, $p_1 = 0.018$, $p_2 = 0.120$, $w_2 = 0.10$ and approximate plans derived from the master table for $p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$.

N	Exact n	c	$N^* = 1.02N$	Approximate $(n^*+20)/3$	c^*
50	Accept		51	Accept	
70	5	0	71	Accept	
100	5	0	102	10	0
200	10	0	204	10	0
300	10	0	306	25	1
500	25	1	510	25	1
700	30	1	714	30	1
1000	45	2	1020	45	2
2000	65	3	2040	65	3
3000	80	4	3060	80	4
5000	100	5	5100	100	5
7000	100	5	7140	100	5
10000	120	6	10200	120	6
20000	135	7	20400	155	8
30000	155	8	30600	155	8
50000	175	9	51000	175	9
70000	190	10	71400	195	10
100000	210	11	102000	210	11
200000	225	12	204000	230	12

As an example consider the determination of n for N = 50,000 and the parameters given in Table 7. The value of $1/\varphi$ is 293 for $p_1 = 0.006$ and $p_2 = 0.040$, and $(1 - \frac{1}{3 \log N})$ = 0.929, so that we find

$$n = \frac{1}{3}(505 + 293(\log 3 + 0.61 \log 2 - \log 3.78)0.929)$$

$$= \frac{1}{3}(505 + 23) = 176$$

in agreement with the (rounded) exact solution, n = 175, given in Table 7.

12. Efficiency.

In a previous paper [6] it has been proposed to define the efficiency of a sampling plan as

$$e(N, n, c) = R_o(N)/R(N, n, c) \qquad (107)$$

where $R_o(N)$ denotes the costs of the optimum plan and $R(N, n, c)$ denotes the costs of the plan in question.

We shall first discuss the efficiency of a sampling plan on the assumption that the optimum relationship between n and c has been used so that the loss in efficiency is due to using a wrong relationship between N and n. Looking at Fig. 2 it will be seen that it does not matter much whether we use the value of c giving the absolute minimum of R or a neighbouring value of c provided n is chosen such that a (relative) minimum of R is obtained.

For a given set of parameters let (n_o, c_o) be optimum for N_o and (n_1, c_1) be optimum for N_1. From (26) it follows that

$$R(N, n, c) = n + (N - n)h(n, c)$$

where

$$h(n, c) = \gamma_1 Q(p_1) + \gamma_2 P(p_2).$$

Using the plan (n_1, c_1) for lot size N_o (instead of N_1) we find

$$R(N_o, n_1, c_1) = n_1 + (N_o - n_1)h(n_1, c_1)$$

$$= n_1 + (N_o - n_1)(R_o(N_1) - n_1)/(N_1 - n_1). \quad (108)$$

It is therefore rather simple by means of the function $R_o(N)$ to evaluate the efficiency of plans contained in the master table in case such plans are used for the wrong value of N. The resulting efficiency is

$$e(N_o, n_1, c_1) = \frac{R_o(N_o)}{n_1 + (N_o - n_1)(R_o(N_1) - n_1)/(N_1 - n_1)} \quad (109)$$

Since $R_o(N) \sim n + 1/\varphi_o$ we have asymptotically

$$e(N_o, n_1, c_1) \sim \left(n_o + \frac{1}{\varphi_o}\right) \Big/ \left(n_1 + \frac{N_o - n_1}{N_1 - n_1}\,\frac{1}{\varphi_o}\right). \quad (110)$$

Introducing $n_o = (\ln N_o)/\varphi_o + o(\ln N_o)$ and considering n_1 as an arbitrary function of N_o, $n_1 = g(N_o)/\varphi_o$ say, we find

$$e(N, n_1, c_1) \sim (\ln N) \Big/ \left(g(N) + Ne^{-g(N)}\right) \quad (111)$$

for $n_1 = o(N)$, which is the result given without proof in [6].

For $g(N) = \lambda \ln N$ we get $e \longrightarrow 1/\lambda$ for $\lambda \geq 1$ but $e \longrightarrow 0$ for $0 < \lambda < 1$, i.e. if we use a semilogaritmic relationship between n and N differing from the correct one then it is <u>important to use too large a sample</u>.

For $g(N) = N^{\lambda}$, $\lambda > 0$, we get $e \longrightarrow 0$. A more accurate expression than (111) may be found by using all three term of (61) which leads to

$$e(N_o,n_1,c_1) \sim (n_o + \frac{1}{\varphi_o}) \Big/ \left(n_1 + \frac{1}{\varphi_o}(e^{\varphi_o(n_o-n_1)}\sqrt{\frac{n_o}{n_1}} + \frac{n_o-n_1}{N_1-n_1})\right).$$

This formula is, however, not of direct value because it contains N_1 which is unknown in practice. A simple and practically useful approximation is the following

$$e(N_o,n_1,c_1) \sim (n_o + \frac{1}{\varphi_o}) \Big/ \left(n_1 + \frac{1}{\varphi_o} e^{\varphi_o(n_o-n_1)}\right). \tag{112}$$

This formula will, however, give too large efficiencies for small values of n because the decision loss has been overestimated.

In connection with the various approximations developed in the preceding sections is has repeatedly been stated that the value of c found by using the approximations will normally not deviate more than 1 from the correct value (for $N < 200{,}000$). It is therefore of importance to know the efficiency of a plan for which $|c_1-c_o| = 1$.

If $|c_1-c_o| = a$ (constant) then $|n_1-n_o| = a\beta$ and $e \longrightarrow 1$ for $N_o \longrightarrow \infty$. Expanding the denominator of (112) we find for small values of $\varphi_o a\beta$

$$e(N_o,n_1,c_1) \sim (n_o + \frac{1}{\varphi_o}) \Big/ (n_o + \frac{1}{\varphi_o} + \frac{1}{2}\varphi_o a^2\beta^2) \tag{113}$$

which converges rather fast to 1 for $n_o \longrightarrow \infty$ and $a = 1$. By means of the results of section 6 it will be seen that this asymptotic efficiency (as a function of c_o) is independent of the "quality level".

An example has been given in Table 8. The costs for each optimum plan have been compared with the costs of using a neighbouring plan, i.e. $c_1 = c_o \pm 1$. The efficiency has been compared with the asymptotic efficiency found from (112). It will be seen that the efficiency is larger than 0.90 for $c \geq 2$ and that the asymptotic formula gives too high an efficiency for small N. (N has been chosen as the geometric mean of the smallest and largest N for each c). This conclusion is typical for the cases investigated.

Table 8.

Investigation of efficiency for sampling plans with an acceptance number deviating 1 from the optimum. (e^* = asymptotic efficiency).
$p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$.

N	n_o	c_o	R	n_1	c_1	100e	100e*
145	10	0	53	60	1	72	94
447	60	1	126	10 115	0 2	84 86	94 95
1010	115	2	200	60 170	1 3	90 92	95 96
1900	170	3	265	115 225	2 4	93 95	96 97
3350	225	4	326	170 280	3 5	95 96	96 97
5700	280	5	386	225 335	4 6	96 97	97 98
9530	335	6	444	280 390	5 7	96 97	97 98
15800	390	7	502	335 445	6 8	97 98	97 98
25800	445	8	559	390 500	7 9	97 98	98 98
42100	500	9	616	445 555	8 10	97 98	98 98
68300	555	10	672	500 615	9 11	98 98	98 98

The conversion formulas and tables show how sensitive the solution is to changes of the parameters. A change of w_2 from 0.05 to 0.10, say, means, that N has to be multiplied by a factor of about 1.3 and the corresponding n should be increased by about 30. (In most systems of sampling plans in practical use to-day the same plan is used for a rather large N-interval, the ratio between endpoints usually being 1.5 or larger). As an example consider the case with $p_r = p_s = 0.010$, $p_1 = 0.006$ and $p_2 = 0.040$ as shown in the following table.

Optimum sampling plans.

	$w_2 = 0.05$		$w_2 = 0.10$	
N	n	c	n	c
500	60	1	85	1
1000	115	2	140	2
5000	275	5	305	5
10000	335	6	405	7
50000	505	9	575	10
100000	610	11	635	11

For most lot sizes we find the same value of c and a difference in n of about 25, in other cases the difference in c is 1 and the difference in n correspondingly larger. It is immediately clear that using the plans corresponding to $w_2 = 0.05$ if the true value of w_2 is 0.10 does not lead to an essential loss in efficiency.

The conclusion is that even if the value of w_2 used deviates from the true value by a factor of 2 the method will nevertheless lead to a sampling plan of very high efficiency.

Similar conclusions may be drawn for the other parameters by studying the conversion formulas.

The main reason why changes of p_r and w_2 does not affect the optimum solution seriously is that p_o and φ_o are independent of p_r and w_2.

Since the most important relation in the system is

$$c + \frac{1}{2} = p_o(n - \alpha)$$

it is of importance to know how p_o depends on p_1 and p_2.

From

$$\frac{\partial \ln p_o}{\partial \ln p_1} = \frac{p_o - p_1}{q_1 \ln \frac{q_1}{q_2}} > 0 \text{ and } \frac{\partial \ln p_o}{\partial \ln p_2} = \frac{p_2 - p_1}{q_2 \ln \frac{q_1}{q_2}} > 0$$

it follows that p_o is an increasing function of as well p_1 as p_2. Furthermore we have approximately

$$\frac{\partial \ln p_o}{\partial \ln p_1} + \frac{\partial \ln p_o}{\partial \ln p_2} \cong 1.$$

Within the domain of variation tabulated the first term is on the average 0.35 and the second 0.65.

The coefficient $p_o\alpha$ varies rather slowly with (p_1, p_2).

Table 9.

Efficiencies of plans obtained by using wrong values of p_1, p_2 or both.

$p_r = p_s = 0.010$, $p_1 = 0.006$, $p_2 = 0.040$, $w_2 = 0.05$.

100 p_1	0.6			0.7			0.5			0.6			0.6			0.7			0.5		
100 p_2	4.0			4.0			4.0			3.5			5.0			3.5			5.0		
N	n	c	R	n	c	100e	n	c	100e	n	c	100e	n	c	100e	n	c	100e	n	c	100e
200	15	0	71	20	0	98	10	0	99	5	0	96	20	0	98	15	0	100	20	0	98
500	60	1	135	65	1	100	55	1	100	55	1	100	65	1	100	65	1	100	60	1	100
1000	115	2	199	120	2	100	110	2	100	115	2	100	110	2	100	120	2	100	105	2	99
2000	170	3	270	215	4	97	170	3	100	175	3	100	155	3	98	225	4	96	155	3	98
5000	275	5	372	320	6	96	235	4	98	295	5	98	205	4	92	345	6	94	210	4	94
10000	335	6	450	380	7	98	295	5	96	360	6	97	295	6	90	455	8	91	260	5	89
20000	395	7	532	480	9	97	355	6	94	480	8	95	340	7	82	570	10	89	310	6	84
50000	505	9	637	585	11	97	470	8	96	600	10	94	395	8	73	690	12	89	365	7	74
100000	610	11	718	690	13	95	530	9	93	720	12	92	445	9	65	805	14	87	415	8	66
200000	670	12	799	745	14	97	590	10	89	785	13	82	530	11	58	915	16	86	465	9	58

It follows that p_o is known with a relative error of about the same size as the relative errors of p_1 and p_2.

If the choice of p_1 and p_2 is doubtful then p_1 should be chosen too large and p_2 too small (by about half of the percentage error in p_1) because the two errors will tend to counterbalance one another and thus give the correct p_o. The reason for bringing the two parameters closer together in case of doubt lies also in the fact that φ_o is a decreasing function of p_1 and an increasing function of p_2. Since $n \sim (\log N)/\varphi_o$ the proposed rule will lead to a larger sample size than the optimum one which normally gives a better efficiency than too small a sample. However, with a wrong p_o (and φ_o) the efficiency will tend to zero because the average decision loss becomes $O(N^{\varepsilon})$, $0 < \varepsilon < 1$, instead of $O(1)$ as for the optimum plan.

Table 9 shows the efficiency of using a plan obtained by entering the master table by a wrong value of p_1, p_2 or both. It is assumed that the true values of (p_1, p_2) are (0.006,0.040) and optimum plans have been substituted by plans obtained by using neighbouring values of (p_1, p_2) in the tables, i.e. the relative error of p_1 is 17% and the relative error of p_2 is 12.5% downwards and 25% upwards. The table shows that the efficiency in all cases is larger than 90% for $N < 10{,}000$. For $N = 200{,}000$, however, the efficiency falls to 58% in the worst case, i.e. the case where p_2 is chosen 25% too large.

The results in the table support the statement above that in case of doubt it is important to use a large value of p_1 and a small value of p_2.

13. An example.

Consider now an example starting from the original cost functions. To show the various aspects of the method the example will be worked out in more detail than is necessary for routine applications.

Let the three cost functions be $k_s(p) = 23 + 35p$, $k_r(p) = 16 + 35p$, $k_a(p) = 720p$, the coefficients denoting costs per item in cents, say, i.e. the costs of sampling and testing is 23 cents per item in the sample and the costs of accepting a defective item is 720 cents etc., see section 2.

Let us further assume that lots are generated with probability $w_1 = 0.93$ from a binomially controlled process with $p_1 = 0.009$ and with probability $w_2 = 0.07$ from a process with $p_2 = 0.080$.

The costs may then be described as in the following table:

w	p	$k_s(p)$	$k_r(p)$	$k_a(p)$	$k_m(p)$	$\lvert k_r(p)-k_a(p)\rvert$
0.93	0.009	23.315	16.315	6.480	6.480	9.835
0.07	0.080	25.800	18.800	57.600	18.800	38.800
Average	0.014	23.489	16.489	10.058	7.342	11.863

From (12) we find

$$p_r = (16-0)/(720-35) = 0.0234,$$

from (28)

$$p_m = 0.93 \times 0.009 + 0.07 \times 0.0234 = 0.0100,$$

and from (22)

$$p_s = (23-0)/(720-35) = 0.0336.$$

To find the optimum plan for N = 500 from the master table with $p_r = p_s = 0.010$ we first have to find the conversion factor λ_s which corrects for the difference between p_s and p_r, i.e.

$$\lambda_s^{-1} = 1 + \frac{336-234}{0.93(234-90)} = 1.76.$$

To use the method of section 8 we find $\gamma_2 = 0.296$, $\lambda = 1.97$, $p_1/\lambda = 0.0046 \cong 0.005$, $p_2/\lambda = 0.041 \cong 0.040$, and $N^* = 1.97\ N/1.76 = 1.12\ N = 560$. From the master table we read $(n^*, c^*) = (60,1)$ which gives $n = 60/1.97 = 30$ as the optimum sample size.

To illustrate the method of section 11 we have to find the conversion factor f_1 and the correction g_1 corresponding to the change from $w_2 = 0.05$ to 0.07. Since $\rho_1 = 90/234 = 0.385 \cong 0.40$ and $\rho_2 = 800/234 = 3.42 \cong 3.50$ we have $f_1 = 1.13$ and $g_1 = 15$. We then enter the master table with

$$N^* = N \times 2.34 \times 1.13/1.76 = 1.50N = 750$$

and find $(n^*, c^*) = (60,1)$ which finally gives $(n,c) = (30,1)$ since $(60 + 15)/2.34 = 32$.

To find the corresponding value of R we first compute

$$\gamma_1 = 0.93(234-90)/(336-100) = 0.567$$

and

$$\gamma_2 = 0.07(800-234)/(336-100) = 0.168$$

which lead to

$$R = n + (N-n)(0.567Q(p_1) + 0.168P(p_2)).$$

From a table of the binomial distribution one finds for $(n,c) = (30,1)$ that $Q(p_1) = 0.02982$ and $P(p_2) = 0.29579$ and consequently

$$R = 30 + 470 \times 0.0666 = 61.3.$$

The costs of sampling inspection and the average decision losses per lot are thus of nearly the same size.

Returning to the original monetary unit we find

$$k - k_m = R(k_s - k_m)/500 = 1.98$$

and finally

$$k = 7.34 + 1.98 = 9.32.$$

We thus have the following conclusion:

The quality of submitted lots is such that on the average costs per item will be 7.34 cents if all lots are classified correctly, i.e. all lots from process No. 1 are accepted and all lots from process No. 2 are rejected. To decide whether to accept or reject we inspect a sample of 30 items at the average costs of 0.97 cents per item of the lot. The decision losses will be 1.01 cents per item of the lot on the average. The first part of the costs, 7.34, depends on the prior distribution and can only be reduced by producing (or buying) lots of better quality. The second part, 1.98, depends on the sampling plan used. Since we have here used the optimum plan any change in sample size or acceptance number will result in increased costs. The average costs of accepting all lots without inspection is 10.06 cents per item.

The two functions $k_o(p) = K_o(p)/N$ and $k(p) = K(p)/N$ have been shown in Fig. 1 for the example above.

14. General remarks.

There exists already a great body of theories and tables for constructing single sampling attribute plans based on <u>two specified quality levels</u> (p_1, p_2) and some further requirements. To see how the present paper fits into this the most important systems have been listed below by stating the "further requirements" for each system:

(a). Specification of the producer's and the consumer's risks, see for instance Peach and Littauer [7] and Grubbs [8].

(b). Specification of the consumer's risk and minimization of the average amount of inspection for lots of process average quality (p_1) in the case of rectifying inspection, see Dodge and Romig [9].

(c). Specification of the consumer's risk and minimization of the average costs for lots of process average quality (p_1), i.e. a generalization of the Dodge-Romig LTPD system requirering specification of one cost parameter, see Hald [10].

(d). Specification of two cost parameters, p_r and p_s, and a weight, w_2, and minimization of the average costs, as for instance in the present paper.

It follows from the results of the present paper that from an economic point of view it is not advisable to fix the consumer's or the producer's risk. On the contrary the producer's and the consumer's risks should both tend to zero with increasing lot size. This theorem is valid not only for the double binomial prior distribution but for any prior distribution, and it is valid not only for the Bayes solution but also for the minimax solution ($p_1 < p_r < p_2$), the only difference being the speed of the convergence. For a discrete prior distribution the risks tend to zero inversely proportional to N, see (63) and (64). These considerations lead to the result that if one wants a system with a fixed risk then the risk should be fixed to 50 per cent at a point between p_1 and p_2. We may therefore increase the list of systems of sampling plans above by the following item:

(e). Minimization of average costs for lots of process average quality (p_1) under the restriction that $P(p_o) = 1/2$. Such a system, named the IQL system (Indifference Quality Level) has been discussed by Hald, see [6] and [10], and will be further discussed in a forthcoming paper. This system requires the specification of p_o and a cost parameter. In view of the asymptotic relation (66) between c and n it is clear that p_o should be determined from (52).

The simplest possible system based on the specification of two risks and having the same properties as the Bayes solution may be formulated as follows:

(f). Specification of the consumer's or the producer's risk as inversely proportional to lot size, and $P(p_o) = 1/2$.

This system requires only the specification of one parameter (besides the two quality levels) and it is extremely simple to handle both mathematically and numerically. This is due to the fact that the equation $P(p_o) = 1/2$ has the solution $c = np_o+(p_o-2)/3$ (with sufficient accuracy for all practical applications, perhaps apart from the case $c = 0$ where the exact solution may be easily found) and that the other equation, $Q(p_1) = \alpha/N$ say, may be solved with respect to N for related values of (n,c) from the first equation. Setting $c = 0,1,2,...$ and solving the first equation for n, the second equation gives $N = \alpha/(1-B(c,n,p_1))$ which may easily be found by means of a table of the binomial (or the Poisson) distribution. The only difficulty lies in the choice of α. If the problem is fully specified one may naturally choose α as the coefficient of 1/N in (63) and the system will then asymptotically give an approximation to the Bayes solution. The reason for using the simple system will,

however, usually be that some of the parameters in the problem are unknown and in that case the choice of α will to some extent be arbitrary, just as in the other cases the choice of the producer's or the consumer's risk is arbitrary. This system of sampling plans will be discussed in more detail in the forthcoming paper on the IQL system.

Turning to applications it is important to notice that a system of sampling plans in practice often is required to serve several purposes. In particular we shall here stress (a) that the system should protect the consumer against deterioration of the prior distribution, (b) that the system should work as an incentive for the producer to produce better quality or at least to keep to the quality agreed upon, see Hill [11], and (c) that (average) costs should be minimized. The first two requirements are concerned with consequences of changes of the prior distribution and the problem should therefore really be formulated as a dynamic one. However, since a dynamic model at present is lacking we shall try to indicate how the Bayesian solution may be modified to take requirements (a) and (b) into account.

One of the arguments advanced against the Bayesian method in general has been that a prior distribution does normally not exist. This may be true in many fields but certainly not for industrial mass production with its effective planning and control of operations. Admittedly the prior distribution may change, but changes are usually rather small and slow within a given production period in which the same machinery, techniques, and raw materials are being used. We are here not concerned about isolated very poor lots which may occasionally occur since any sampling plan will detect such lots.

Published data on prior distributions are scarce. Whether the double binomial distribution is a reasonable approximation to distributions occurring in practice is not known. According to the experience of the author mixed binomial distributions with beta-distributions as weight functions are rather common. (A paper analogous to the present one will present the corresponding theory and tables for the beta-distribution).

One of the drawbacks of the Bayesian solution from a practical point of view is that the solution may be acceptance (or rejection) without inspection. If one is not completely confident that the prior distribution used is the right one and is stable, then a sampling plan is required to guard against deterioration of the prior distribution. One possibility is to use the first or one of the first Bayesian sampling plans in the appropriate table. If that is not satisfactory one may in such cases use an IQL plan.

The same procedure may be used to satisfy requirement (a) above. It should first of all be noted that if a Bayesian sampling plan exists then some protection against deterioration of the prior distribution is automatically obtained and the protection may in the usual way be expressed by means of the OC curve. It is always easy when the plan has been found to compute the consumer's risk and then to decide whether the risk is sufficiently small. If the consumer's risk is too large one may again find a sufficiently large sample in the same table or turn to an IQL plan or a LTPD plan.

The price to be paid for obtaining the required protection is naturally that the plan used will not minimize costs if the prior distribution holds. If the change in the value of c is not large the increase in costs will, however, be small.

For large lots the consumer's risk for the Bayesian sampling plan will usually be much smaller than 10 per cent so that the problem does only exist for small lots.

The incentive for the producer to keep to the specified quality is usually obtained by alternating between normal and tightened inspection in a specific way such that the system reacts upon observed changes in the prior distribution. If it was possible to estimate in what way the distribution had changed the reaction could be made to depend on the change. In practice, however, one wants to install tightened inspection as soon as possible on the basis of some over-all criterion, for example when the number of lots rejected exceeds some critical limit. A thorough theory does not exist but some rules have been found to work satisfactory in practice. The Military Standard 105 D uses the same sample size for normal and tightened inspection and a reduced acceptance number, c_T, for tightened inspection. The difference between the two acceptance numbers, c_N-c_T, equals 1 for $2 \leq c_N \leq 4$, 2 for $5 \leq c_N \leq 20$, and 3 for $c_N \geq 21$. For $c_N = 0$ or 1, c_T is usually equal to c_N but the sample size is increased for tightened inspection. Similar rules may be used for the present tables although it has to be realized that the resulting plans will not be minimum-cost plans. The main point is, that under normal conditions the plans will minimize costs and that the plans may be adjusted to changes in the prior distribution so that costs are minimized under the new conditions. If, however, the incentive aspect of sampling inspection is more important for the user of the system than to minimize costs in case of change of the distribution then some form of tightened inspection may be introduced with the result that during periods of tightened inspection the plans will not minimize costs.

Acknowledgements.

This paper has been on its way for a period of five years and I owe a debt of gratitude to the many collaborators I have had. In particular I wish to thank Mr. J. Vestergaard for making the first program for tabulating the exact solution which resulted in a basic set of tables of great value for all the subsequent analyses. Mr. Vestergaard also checked the accuracy of the first asymptotic expression. I am also most grateful to Mr. and Mrs. K. West Andersen who carried out the detailed numerical investigations of the asymptotic formulas, tabulated the conversion factors, made the final program for the tables, and checked the whole manuscript.

The tabulation was made possible by a grant from the Carlsberg Foundation.

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Appendix.

Master Tables of Sampling Plans.

Tables of Conversion Factors.

Summary of Conversion Formulas.

All sampling plans in the master tables assume $p_r = p_s$ and $w_2 = 0.05$.

In the first set of tables $p_r = 10$ % and (p_1, p_2) take on the values

p_1 = 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, 6.0, 7.0 %

p_2 = 15.0, 17.5, 20.0, 25.0, 30.0 %.

In the second set of tables $p_r = 1$ % and (p_1, p_2) take on the values

p_1 = 0.20, 0.25, 0.30, 0.35, 0.40, 0.50, 0.60, 0.70 %

p_2 = 1.50, 1.75, 2.00, 2.50, 3.00, 3.50, 4.00, 5.00, 6.00, 7.00 %.

Single Sampling Tables for $p_1 = 2.0$ %

$p_2 = 15.0$ %

N	n (Accept)	c
1 – 1460		
1460 – 1530	16	2
1600 – 1840	29	3
2160 – 2630	31	3
2630 – 2760	43	4
3180 – 3760	45	4
4560 – 4770	58	5
5530 – 6550	60	5
7990 – 9560	74	6
11300 – 14000	76	6
14000 – 16400	89	7
19500 – 24300	91	7
24300 – 28200	104	8
33300 – 40300	106	8
42500 – 48000	119	9
56700 – 68400	121	9
73800 – 81600	134	10
96200 – 116000	136	10
128000 – 138000	148	11
163000 – 200000	151	11

$p_2 = 17.5$ %

N	n (Accept)	c
1 – 665		
666 – 701	16	2
829 – 1020	18	2
1170 – 1270	29	3
1510 – 1850	31	3
2270 – 2710	43	4
3280 – 4100	45	4
4410 – 4810	56	5
5760 – 7080	58	5
8530 – 10000	70	6
12200 – 15200	72	6
16400 – 17400	83	7
20900 – 25700	85	7
31300 – 35800	97	8
43400 – 54100	99	8
59700 – 73500	111	9
90200 – 113000	113	9
113000 – 125000	124	10
151000 – 188000	126	10

$p_2 = 20.0$ %

N	n (Accept)	c
1 – 334		
335 – 414	7	1
479 – 586	17	2
727 – 946	19	2
1020 – 1240	29	3
1540 – 1980	31	3
2200 – 2530	41	4
3120 – 3990	43	4
4690 – 5040	53	5
6200 – 7850	55	5
9860 – 12100	66	6
15300 – 20400	68	6
20400 – 23600	78	7
29400 – 37800	80	7
42500 – 45600	90	8
56400 – 71700	92	8
87600 – 108000	103	9
136000 – 178000	105	9
178000 – 200000	115	10

$p_2 = 25.0$ %

N	n (Accept)	c
1 – 140		
141 – 149	6	1
191 – 259	8	1
358 – 396	16	2
509 – 682	18	2
945 – 1200	27	3
1590 – 2240	29	3
2380 – 2720	37	4
3560 – 4880	39	4
5910 – 7790	48	5
10500 – 14200	50	5
14200 – 16800	58	6
22200 – 30800	60	6
34600 – 47100	69	7
63800 – 81900	71	7
81900 – 99600	79	8
132000 – 200000	81	8

$p_2 = 30.0$ %

N	n (Accept)	c
1 – 45		
46 – 92	1	0
93 – 103	6	1
139 – 199	8	1
316 – 346	15	2
471 – 679	17	2
974 – 1400	25	3
2030 – 2800	27	3
2800 – 3970	34	4
5750 – 7850	36	4
7850 – 11000	43	5
15900 – 21600	45	5
21600 – 29800	52	6
43000 – 58900	54	6
58900 – 80000	61	7
115000 – 159000	63	7
159000 – 200000	70	8

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 2.5$ %

$p_2 = 15.0$ %

N	n	c
1 – 1600	Accept	
1600 – 1870	26	3
2050 – 2440	39	4
2930 – 3120	41	4
3120 – 3230	52	5
3840 – 4640	54	5
4860 – 5150	66	6
6060 – 7330	68	6
7620 – 8070	80	7
9520 – 11500	82	7
12000 – 12600	94	8
14900 – 18100	96	8
18700 – 19700	108	9
23300 – 28200	110	9
29400 – 30600	122	10
36200 – 43900	124	10
46000 – 56200	137	11
68100 – 71800	139	11
71800 – 87000	151	12
105000 – 112000	153	12
112000 – 134000	165	13
163000 – 174000	167	13
174000 – 200000	179	14

$p_2 = 17.5$ %

N	n	c
1 – 698	Accept	
699 – 791	15	2
944 – 1040	26	3
1250 – 1610	28	3
1610 – 1960	39	4
2410 – 2780	41	4
2780 – 3050	51	5
3700 – 4790	53	5
4790 – 5670	64	6
6980 – 8250	66	6
8250 – 8690	76	7
10500 – 13200	78	7
14100 – 15900	89	8
19500 – 24000	91	8
24000 – 29100	102	9
36100 – 40800	104	9
40800 – 43700	114	10
53300 – 68800	116	10
68800 – 79300	127	11
97700 – 117000	129	11
117000 – 144000	140	12
179000 – 200000	142	12

$p_2 = 20.0$ %

N	n	c
1 – 367	Accept	
368 – 417	6	1
418 – 485	15	2
605 – 777	17	2
778 – 876	26	3
1090 – 1410	28	3
1490 – 1890	38	4
2410 – 2810	40	4
2810 – 3220	49	5
4050 – 5260	51	5
5260 – 6750	61	6
8680 – 9750	63	6
9750 – 11200	72	7
14200 – 18000	74	7
18000 – 23200	84	8
29900 – 33000	86	8
33000 – 37900	95	9
48000 – 60400	97	9
60400 – 77600	107	10
100000 – 110000	109	10
110000 – 126000	118	11
159000 – 200000	120	11

$p_2 = 25.0$ %

N	n	c
1 – 139	Accept	
140 – 161	6	1
221 – 291	8	1
292 – 347	15	2
459 – 673	17	2
674 – 867	25	3
1170 – 1520	27	3
1520 – 2070	35	4
2860 – 3290	37	4
3290 – 3660	44	5
4850 – 7000	46	5
7000 – 8350	54	6
11200 – 15000	56	6
15000 – 18900	64	7
25700 – 31700	66	7
31700 – 42500	74	8
58700 – 66100	76	8
66100 – 71800	83	9
95300 – 137000	85	9
137000 – 159000	93	10

$p_2 = 30.0$ %

N	n	c
1 – 51	Accept	
52 – 84	1	0
85 – 105	6	1
150 – 246	8	1
247 – 276	14	2
382 – 570	16	2
670 – 856	23	3
1230 – 1690	25	3
1690 – 1840	31	4
2580 – 4100	33	4
4100 – 5330	40	5
7730 – 10200	42	5
10200 – 15400	49	6
23800 – 30900	57	7
45000 – 57800	59	7
57800 – 87700	66	8
133000 – 173000	74	9

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for p_1 = 3.0 %

p_2 = 15.0 %

N	n	c
1 – 1760	Accept	
1760 – 2100	35	4
2420 – 2960	48	5
3450 – 4180	61	6
4960 – 5910	74	7
7160 – 8350	87	8
10300 – 11800	100	9
14200 – 15000	102	9
15000 – 16500	113	10
20000 – 21700	115	10
21700 – 23200	126	11
27900 – 31400	128	11
31400 – 39000	140	12
45200 – 54400	153	13
65000 – 75800	166	14
93300 – 106000	179	15
128000 – 134000	181	15
134000 – 147000	192	16
178000 – 193000	194	16
193000 – 200000	205	17

p_2 = 17.5 %

N	n	c
1 – 749	Accept	
750 – 823	14	2
824 – 980	24	3
1250 – 1340	35	4
1630 – 1990	37	4
1990 – 2200	47	5
2710 – 3150	49	5
3150 – 3600	59	6
4460 – 4970	61	6
4970 – 5840	71	7
7300 – 7820	73	7
7820 – 9440	83	8
12200 – 15200	95	9
19100 – 24400	107	10
29700 – 31700	118	11
39100 – 46300	120	11
46300 – 50400	130	12
62500 – 71800	132	12
71800 – 80000	142	13
99600 – 111000	144	13
111000 – 127000	154	14
159000 – 172000	156	14
172000 – 200000	166	15

p_2 = 20.0 %

N	n	c
1 – 396	Accept	
397 – 481	14	2
636 – 730	24	3
923 – 1110	26	3
1110 – 1340	35	4
1730 – 1920	37	4
1920 – 2440	46	5
3250 – 3470	56	6
4370 – 5550	58	6
5550 – 6120	67	7
7760 – 9380	69	7
9380 – 10700	78	8
13700 – 15800	80	8
15800 – 18700	89	9
24200 – 26400	91	9
26400 – 32600	100	10
44000 – 56500	111	11
73100 – 77200	121	12
97800 – 121000	123	12
121000 – 133000	132	13
169000 – 200000	134	13

p_2 = 25.0 %

N	n	c
1 – 138	Accept	
139 – 178	6	1
248 – 299	14	2
404 – 523	16	2
524 – 602	23	3
807 – 1060	25	3
1060 – 1150	32	4
1530 – 2110	34	4
2110 – 2830	42	5
4080 – 5140	51	6
7120 – 7910	53	6
7910 – [illegible]280	60	7
12700 – 15200	62	7
15200 – 16600	69	8
22400 – 29000	71	8
29000 – 39500	79	9
54600 – 69500	88	10
97000 – 103000	90	10
103000 – 122000	97	11
168000 – 194000	99	11
194000 – 200000	106	12

p_2 = 30.0 %

N	n	c
1 – 57	Accept	
58 – 78	1	0
79 – 109	6	1
167 – 209	8	1
210 – 298	14	2
454 – 503	16	2
504 – 708	22	3
1130 – 1590	30	4
2520 – 3460	38	5
5500 – 7410	46	6
11100 – 12000	48	6
12000 – 15700	54	7
23500 – 25700	56	7
25700 – 33000	62	8
49100 – 54800	64	8
54800 – 68800	70	9
102000 – 116000	72	9
116000 – 143000	78	10

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 3.5$ %

$p_2 = 15.0$ %

N	n	c
1 – 1850	Accept	
1850 – 2030	32	4
2030 – 2150	43	5
2580 – 2690	45	5
2690 – 3210	56	6
3620 – 4040	68	7
4910 – 6120	81	8
6660 – 7660	93	9
9030 – 9610	105	10
11600 – 12300	107	10
12300 – 14500	118	11
16700 – 18100	130	12
22600 – 27200	143	13
30700 – 33800	155	14
41500 – 50800	168	15
56200 – 62900	180	16
76200 – 94600	193	17
103000 – 117000	205	18
139000 – 175000	218	19
187000 – 200000	230	20

$p_2 = 17.5$ %

N	n	c
1 – 787	Accept	
788 – 906	22	3
1060 – 1290	33	4
1550 – 1840	44	5
2270 – 2610	55	6
3340 – 3670	66	7
4580 – 4960	68	7
4960 – 6360	78	8
7260 – 8830	89	9
10600 – 12200	100	10
15400 – 16900	111	11
21200 – 22600	113	11
22600 – 29000	123	12
32800 – 39700	134	13
47600 – 54400	145	14
68700 – 74600	156	15
93600 – 100000	158	15
100000 – 127000	168	16
144000 – 173000	179	17

$p_2 = 20.0$ %

N	n	c
1 – 399	Accept	
400 – 470	13	2
549 – 591	22	3
746 – 886	24	3
887 – 1150	33	4
1420 – 1750	43	5
2250 – 2620	53	6
3400 – 3570	55	6
3570 – 3890	63	7
4990 – 5660	65	7
5660 – 7300	74	8
8840 – 10700	84	9
13800 – 15500	94	10
20100 – 21600	96	10
21600 – 28900	105	11
33400 – 41600	115	12
51600 – 59900	125	13
79400 – 86400	135	14
112000 – 123000	137	14
123000 – 159000	146	15
190000 – 200000	156	16

$p_2 = 25.0$ %

N	n	c
1 – 138	Accept	
139 – 215	6	1
216 – 252	13	2
342 – 426	15	2
427 – 531	22	3
741 – 804	24	3
805 – 1070	31	4
1480 – 2110	40	5
2660 – 2990	48	6
4110 – 4760	50	6
4760 – 5680	57	7
7930 – 8430	59	7
8430 – 10700	66	8
14900 – 20200	75	9
26400 – 38000	84	10
45700 – 51700	92	11
71400 – 79600	94	11
79600 – 95600	101	12
138000 – 177000	110	13

$p_2 = 30.0$ %

N	n	c
1 – 62	Accept	
63 – 71	1	0
72 – 80	5	1
115 – 175	7	1
176 – 227	13	2
340 – 398	15	2
399 – 569	21	3
833 – 940	28	4
1370 – 1710	30	4
1710 – 2140	36	5
3230 – 3450	38	5
3450 – 4850	44	6
6950 – 11000	52	7
13600 – 16600	59	8
24900 – 26600	61	8
26600 – 36500	67	9
52400 – 80700	75	10
101000 – 120000	82	11
180000 – 196000	84	11
196000 – 200000	90	12

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 4.0\ \%$

$p_2 = 15.0\ \%$			$p_2 = 17.5\ \%$			$p_2 = 20.0\ \%$			$p_2 = 25.0\ \%$			$p_2 = 30.0\ \%$		
N	n	c Accept	N	n	c Accept	N	n	c Accept	N	n	c Accept	N	n	c Accept
1 – 1950			1 – 806			1 – 402			1 – 134			1 – 65		
1950 – 2130	40	5	807 – 923	21	3	403 – 447	12	2	135 – 151	5	1	66 – 79	5	1
2260 – 2850	52	6	924 – 981	30	4	487 – 576	21	3	198 – 276	13	2	119 – 156	7	1
2850 – 3090	63	7	1260 – 1470	41	5	736 – 944	31	4	359 – 456	21	3	157 – 244	13	2
3670 – 4130	75	8	1770 – 2210	52	6	1110 – 1190	40	5	632 – 719	29	4	326 – 442	20	3
4730 – 5520	87	9	2470 – 2650	62	7	1530 – 1670	42	5	996 – 1100	31	4	640 – 760	27	4
6120 – 7390	99	10	3420 – 3930	73	8	1670 – 1880	50	6	1100 – 1490	38	5	1140 – 1240	29	4
7920 – 10200	111	11	4780 – 5830	84	9	2490 – 2950	60	7	1860 – 2220	46	6	1240 – 1860	35	5
10200 – 10800	122	12	6660 – 8660	95	10	3730 – 4610	70	8	3130 – 4530	55	7	2320 – 3010	42	6
13200 – 14400	134	13	9170 – 10100	105	11	5550 – 7190	80	9	5210 – 6550	63	8	4300 – 4870	49	7
17000 – 19100	146	14	12700 – 14900	116	12	8210 – 11200	90	10	8620 – 9530	71	9	7280 – 7980	51	7
22000 – 25500	158	15	17500 – 21900	127	13	12000 – 13200	99	11	13300 – 14200	73	9	7980 – 11400	57	8
28400 – 33900	170	16	24100 – 25500	137	14	17500 – 20300	109	12	14200 – 18900	80	10	14600 – 18000	64	9
36600 – 45100	182	17	33000 – 37200	148	15	25700 – 31100	119	13	23400 – 27100	88	11	26800 – 42500	72	10
47200 – 60400	194	18	45500 – 54300	159	16	37700 – 47600	129	14	38300 – 53700	97	12	48500 – 65700	79	11
60400 – 64700	205	19	62600 – 79400	170	17	55200 – 73000	139	15	62500 – 76100	105	13	87600 – 103000	86	12
77600 – 85600	217	20	85500 – 91700	180	18	79900 – 85600	148	16	103000 – 151000	114	14	158000 – 200000	94	13
100000 – 113000	229	21	117000 – 133000	191	19	115000 – 130000	158	17	165000 – 200000	122	15			
129000 – 150000	241	22	160000 – 200000	202	20	168000 – 200000	168	18						
165000 – 200000	253	23												

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for p_1 = 5.0 %

p_2 = 15.0 %

N	n	c
1 - 2090	Accept	
2090 - 2110	45	6
2110 - 2240	55	7
2480 - 2750	66	8
2950 - 3400	77	9
3520 - 4210	88	10
4210 - 5050	99	11
5050 - 5230	109	12
6060 - 6460	120	13
7270 - 7970	131	14
8730 - 9850	142	15
10500 - 12200	153	16
12600 - 15000	164	17
15000 - 18100	175	18
18100 - 21600	186	19
21600 - 22700	196	20
25900 - 27900	207	21
30900 - 34300	218	22
37000 - 42200	229	23
44200 - 52800	240	24
52800 - 63200	251	25
63200 - 75700	262	26
75700 - 90100	273	27
90100 - 94900	283	28
108000 - 116000	294	29
128000 - 143000	305	30
153000 - 175000	316	31
183000 - 200000	327	32

p_2 = 17.5 %

N	n	c
1 - 822	Accept	
823 - 949	27	4
950 - 1000	36	5
1210 - 1340	46	6
1560 - 1790	56	7
2000 - 2390	66	8
2570 - 3280	76	9
3280 - 4210	86	10
4210 - 5350	96	11
5350 - 5750	105	12
6820 - 7550	115	13
8680 - 9900	125	14
11000 - 13000	135	15
14000 - 17000	145	16
17700 - 22400	155	17
22400 - 23400	165	18
28400 - 29600	174	19
35900 - 38600	184	20
45300 - 50200	194	21
57200 - 65300	204	22
72200 - 84900	214	23
91000 - 111000	224	24
115000 - 144000	234	25
144000 - 181000	244	26
181000 - 189000	253	27

p_2 = 20.0 %

N	n	c
1 - 404	Accept	
405 - 506	19	3
553 - 716	28	4
766 - 998	37	5
1060 - 1370	46	6
1450 - 1860	55	7
1980 - 2510	64	8
2690 - 3360	73	9
3650 - 4480	82	10
4920 - 5950	91	11
6630 - 7880	100	12
8910 - 10400	109	13
11900 - 13700	118	14
16000 - 18000	127	15
21400 - 23700	136	16
28500 - 31100	145	17
38000 - 40700	154	18
50900 - 67500	164	19
67500 - 89600	173	20
89600 - 119000	182	21
119000 - 157000	191	22
157000 - 200000	200	23

p_2 = 25.0 %

N	n	c
1 - 123	Accept	
124 - 160	5	1
161 - 230	12	2
268 - 304	19	3
432 - 526	27	4
685 - 888	35	5
1060 - 1480	43	6
1630 - 2420	51	7
2420 - 2770	58	8
3660 - 4450	66	9
5500 - 7130	74	10
8230 - 11400	82	11
12300 - 17900	90	12
17900 - 20300	97	13
26600 - 32000	105	14
39400 - 50500	113	15
58200 - 80000	121	16
85700 - 124000	129	17
124000 - 139000	136	18
183000 - 200000	144	19

p_2 = 30.0 %

N	n	c
1 - 56	Accept	
57 - 74	5	1
121 - 176	12	2
236 - 359	19	3
424 - 713	26	4
714 - 825	32	5
1220 - 1470	39	6
2050 - 2600	46	7
3420 - 4570	53	8
5650 - 7990	60	9
9290 - 14000	67	10
15200 - 24300	74	11
24300 - 27300	80	12
39700 - 46700	87	13
64500 - 79700	94	14
105000 - 136000	101	15
169000 - 200000	108	16

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 6.0\ \%$

$p_2 = 15.0\ \%$

N	n	c
1 – 2110	Accept	
2110 – 2160	59	8
2170 – 2390	69	9
2450 – 2670	79	10
2770 – 2980	89	11
3160 – 3340	99	12
3600 – 3730	109	13
4110 – 4680	120	14
4680 – 5330	130	15
5330 – 6070	140	16
6070 – 6920	150	17
6920 – 7880	160	18
7880 – 8960	170	19
8960 – 10200	180	20
10200 – 11200	190	21
11600 – 12500	200	22
13300 – 13900	210	23
15100 – 15500	220	24
17200 – 19600	231	25
19600 – 22200	241	26
22200 – 25200	251	27
25200 – 28600	261	28
28600 – 32500	271	29
32500 – 36800	281	30
36800 – 40600	291	31
41900 – 45000	301	32
47600 – 49900	311	33
54100 – 55400	321	34
61500 – 69700	332	35
69700 – 79000	342	36
79000 – 89400	352	37
89400 – 101000	362	38
101000 – 115000	372	39

Continued ----->

$p_2 = 17.5\ \%$

N	n	c
1 – 784	Accept	
785 – 872	33	5
917 – 1020	42	6
1110 – 1190	51	7
1350 – 1630	61	8
1630 – 1970	70	9
1970 – 2380	79	10
2380 – 2760	88	11
2870 – 3170	97	12
3470 – 3640	106	13
4190 – 5010	116	14
5010 – 6000	125	15
6000 – 7160	134	16
7160 – 7980	143	17
8630 – 9080	152	18
10300 – 12300	162	19
12300 – 14700	171	20
14700 – 17500	180	21
17500 – 19500	189	22
20900 – 22100	198	23
25000 – 29700	208	24
29700 – 35300	217	25
35300 – 41900	226	26
41900 – 46700	235	27
50100 – 52700	244	28
59700 – 70800	254	29
70800 – 83900	263	30
83900 – 99300	272	31
99300 – 110000	281	32
118000 – 124000	290	33
141000 – 167000	300	34
167000 – 200000	309	35

$p_2 = 20.0\ \%$

N	n	c
1 – 361	Accept	
362 – 435	18	3
436 – 563	26	4
564 – 679	34	5
737 – 788	42	6
955 – 1220	51	7
1220 – 1350	59	8
1580 – 2000	68	9
2000 – 2260	76	10
2550 – 3210	85	11
3210 – 3730	93	12
4090 – 5120	102	13
5120 – 6070	110	14
6480 – 8100	119	15
8100 – 10100	127	16
10100 – 10800	135	17
12700 – 15900	144	18
15900 – 17200	152	19
20000 – 24800	161	20
24800 – 27500	169	21
31100 – 38600	178	22
38600 – 43600	186	23
48400 – 60000	195	24
60000 – 69200	203	25
75000 – 92900	212	26
92900 – 109000	220	27
116000 – 144000	229	28
144000 – 177000	237	29
177000 – 187000	245	30

$p_2 = 25.0\ \%$

N	n	c
1 – 103	Accept	
104 – 130	5	1
131 – 161	11	2
208 – 236	18	3
319 – 465	26	4
466 – 608	33	5
677 – 787	40	6
975 – 1360	48	7
1360 – 1890	55	8
1890 – 2260	62	9
2690 – 3700	70	10
3700 – 5060	77	11
5060 – 6150	84	12
7050 – 7590	91	13
9720 – 13200	99	14
13200 – 16300	106	15
18200 – 19900	113	16
25100 – 33900	121	17
33900 – 42500	128	18
46500 – 51600	135	19
63800 – 86100	143	20
86100 – 109000	150	21
117000 – 132000	157	22
161000 – 200000	165	23

$p_2 = 30.0\ \%$

N	n	c
1 – 39	Accept	
40 – 46	1	0
47 – 63	5	1
97 – 111	11	2
175 – 248	18	3
294 – 341	24	4
475 – 693	31	5
739 – 888	37	6
1160 – 1750	44	7
1750 – 2180	50	8
2710 – 4050	57	9
4050 – 5230	63	10
6220 – 9220	70	11
9220 – 12300	76	12
13800 – 15200	82	13
20700 – 28700	89	14
30800 – 35000	95	15
46200 – 67600	102	16
67600 – 80200	108	17
102000 – 149000	115	18
149000 – 183000	121	19

$p_2 = 15.0\ \%$

N	n	c
115000 – 130000	382	40
130000 – 142000	392	41
147000 – 157000	402	42
167000 – 174000	412	43
190000 – 193000	422	44

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 7.0\ \%$

$p_2 = 15.0\ \%$

N	n	c
	Accept	
1 – 1960		
1960 – 2030	73	10
2030 – 2220	82	11
2220 – 2260	91	12
2440 – 2680	101	13
2680 – 2780	110	14
2940 – 3240	120	15
3240 – 3430	129	16
3560 – 3920	139	17
3920 – 4230	148	18
4310 – 4750	158	19
4750 – 5210	167	20
5210 – 5740	177	21
5740 – 6310	186	22
6310 – 6920	196	23
6920 – 7620	205	24
7620 – 8350	215	25
8350 – 9170	224	26
9170 – 9420	233	27
10000 – 11000	243	28
11000 – 11500	252	29
12100 – 13300	262	30
13300 – 14000	271	31
14500 – 15900	281	32
15900 – 17100	290	33
17400 – 19100	300	34
19100 – 20900	309	35
20900 – 22900	319	36
22900 – 25100	328	37
25100 – 27500	338	38
27500 – 30100	347	39

Continued

$p_2 = 17.5\ \%$

N	n	c
	Accept	
1 – 668		
669 – 706	31	5
707 – 735	39	6
823 – 958	48	7
959 – 1120	57	8
1120 – 1220	65	9
1300 – 1520	74	10
1520 – 1760	83	11
1760 – 1970	91	12
2040 – 2360	100	13
2360 – 2450	108	14
2720 – 3130	117	15
3130 – 3610	126	16
3610 – 3820	134	17
4150 – 4760	143	18
4760 – 5470	152	19
5470 – 5880	160	20
6260 – 7190	169	21
7190 – 8220	178	22
8220 – 8990	186	23
9380 – 10800	195	24
10800 – 12300	204	25
12300 – 13700	212	26
14000 – 16000	221	27
16000 – 16400	229	28
18200 – 20700	238	29
20700 – 23600	247	30
23600 – 24800	255	31
27000 – 30800	264	32
30800 – 35100	273	33
35100 – 37200	281	34

Continued

$p_2 = 20.0\ \%$

N	n	c
	Accept	
1 – 287		
288 – 335	17	3
336 – 357	24	4
424 – 462	32	5
532 – 591	40	6
663 – 746	48	7
818 – 934	56	8
1000 – 1160	64	9
1220 – 1430	72	10
1490 – 1800	80	11
1800 – 2170	88	12
2170 – 2620	96	13
2620 – 3150	104	14
3150 – 3780	112	15
3780 – 4540	120	16
4540 – 5430	128	17
5430 – 6490	136	18
6490 – 7760	144	19
7760 – 9260	152	20
9260 – 11000	160	21
11000 – 13100	168	22
13100 – 15600	176	23
15600 – 18600	184	24
18600 – 22100	192	25
22100 – 26300	200	26
26300 – 31200	208	27
31200 – 37000	216	28
37000 – 43800	224	29
43800 – 45200	231	30
51900 – 54000	239	31
61400 – 64600	247	32
72700 – 77100	255	33
86000 – 92000	263	34

Continued

$p_2 = 25.0\ \%$

N	n	c
	Accept	
1 – 73		
74 – 101	5	1
102 – 129	11	2
161 – 211	18	3
236 – 331	25	4
332 – 460	32	5
461 – 627	39	6
628 – 837	46	7
838 – 892	52	8
1110 – 1220	59	9
1470 – 1660	66	10
1930 – 2250	73	11
2520 – 3040	80	12
3290 – 4100	87	13
4280 – 5550	94	14
5550 – 7220	101	15
7220 – 9390	108	16
9390 – 12200	115	17
12200 – 15800	122	18
15800 – 20300	129	19
20300 – 21500	135	20
26100 – 28500	142	21
33500 – 37800	149	22
43100 – 50200	156	23
55300 – 66700	163	24
70900 – 90500	170	25
90500 – 117000	177	26
117000 – 150000	184	27
150000 – 200000	191	28

$p_2 = 30.0\ \%$

N	n	c
	Accept	
1 – 19		
20 – 36	1	0
37 – 45	5	1
76 – 88	11	2
135 – 213	18	3
214 – 324	24	4
325 – 478	30	5
479 – 662	36	6
701 – 912	42	7
1010 – 1240	48	8
1460 – 1690	54	9
2080 – 2280	60	10
2980 – 4180	67	11
4180 – 5860	73	12
5860 – 8180	79	13
8180 – 11400	85	14
11400 – 15100	91	15
16000 – 20000	97	16
22500 – 26400	103	17
31500 – 34900	109	18
44400 – 61500	116	19
61500 – 85100	122	20
85100 – 117000	128	21
117000 – 162000	134	22
162000 – 200000	140	23

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 7.0\ \%$

$p_2 = 15.0\ \%$

N	n	c
00 – 32900	357	40
00 – 36000	366	41
00 – 39300	376	42
00 – 43000	385	43
00 – 44100	394	44
00 – 51500	404	45
00 – 53400	413	46
00 – 61500	423	47
00 – 64700	432	48
00 – 73400	442	49
00 – 78300	451	50
00 – 87600	461	51
00 – 95400	470	52
00 – 104000	480	53
00 – 114000	489	54
00 – 125000	499	55
00 – 136000	508	56
00 – 148000	518	57
00 – 162000	527	58
00 – 177000	537	59
00 – 193000	546	60
00 – 197000	555	61

$p_2 = 17.5\ \%$

N	n	c
39900 – 45500	290	35
45500 – 51800	299	36
51800 – 55700	307	37
58700 – 67000	316	38
67000 – 76100	325	39
76100 – 83300	333	40
86300 – 98600	342	41
98600 – 112000	351	42
112000 – 126000	359	43
126000 – 144000	368	44
144000 – 148000	376	45
164000 – 185000	385	46
185000 – 200000	394	47

$p_2 = 20.0\ \%$

N	n	c
102000 – 110000	271	35
120000 – 131000	279	36
142000 – 156000	287	37
168000 – 186000	295	38

$p_2 = 25.0\ \%$

N	n	c

$p_2 = 30.0\ \%$

N	n	c

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+1,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.20$ %

$p_2 = 1.50$ %

N	n	c Accept
1 – 15700		
00 – 16100	155	2
00 – 16600	275	3
00 – 18700	285	3
00 – 21600	295	3
00 – 25400	305	3
00 – 26300	425	4
00 – 29800	435	4
00 – 34500	445	4
00 – 41000	455	4
00 – 44900	580	5
00 – 51500	590	5
00 – 60000	600	5
00 – 71800	610	5
00 – 77100	735	6
00 – 88800	745	6
00 – 104000	755	6
00 – 122000	765	6
00 – 124000	885	7
00 – 142000	895	7
00 – 165000	905	7
00 – 196000	915	7

$p_2 = 1.75$ %

N	n	c Accept
1 – 7060		
7060 – 7200	155	2
7710 – 8320	165	2
9060 – 9950	175	2
11100 – 11400	185	2
11400 – 11900	285	3
12800 – 13800	295	3
15000 – 16300	305	3
17900 – 19900	315	3
21100 – 22700	425	4
24500 – 26600	435	4
29000 – 31800	445	4
35100 – 39400	455	4
39400 – 40000	560	5
43100 – 46600	570	5
50700 – 55400	580	5
60900 – 67500	590	5
73100 – 75000	700	6
81000 – 87800	710	6
95700 – 105000	720	6
116000 – 128000	730	6
135000 – 140000	840	7
151000 – 164000	850	7
179000 – 200000	860	7

$p_2 = 2.00$ %

N	n	c Accept
1 – 3630		
3630 – 3770	60	1
4210 – 4770	70	1
4770 – 5140	160	2
5550 – 6040	170	2
6620 – 7320	180	2
8190 – 9270	190	2
9670 – 10500	285	3
11400 – 12400	295	3
13700 – 15100	305	3
16900 – 19200	315	3
19800 – 20700	410	4
22600 – 24700	420	4
27200 – 30100	430	4
33600 – 37900	440	4
40200 – 44000	540	5
48100 – 52900	550	5
58500 – 65100	560	5
73200 – 80500	570	5
80500 – 84500	665	6
92400 – 101000	675	6
112000 – 124000	685	6
139000 – 160000	695	6
160000 – 176000	795	7
193000 – 200000	805	7

$p_2 = 2.50$ %

N	n	c Accept
1 – 1510		
1510 – 1640	60	1
1810 – 2020	70	1
2290 – 2640	80	1
3100 – 3440	90	1
3440 – 3710	160	2
4090 – 4550	170	2
5100 – 5780	180	2
6630 – 7720	190	2
8500 – 9260	270	3
10300 – 11500	280	3
13000 – 14800	290	3
17100 – 20100	300	3
20100 – 22200	380	4
24300 – 27800	390	4
31500 – 36000	400	4
41600 – 46500	410	4
46500 – 52000	490	5
58100 – 65500	500	5
74300 – 85200	510	5
99000 – 106000	520	5
106000 – 120000	600	6
135000 – 152000	610	6
173000 – 200000	620	6

$p_2 = 3.00$ %

N	n	c Accept
1 – 508		
509 – 728	5	0
943 – 967	55	1
1070 – 1200	65	1
1360 – 1560	75	1
1820 – 2170	85	1
2640 – 2960	95	1
2960 – 3250	155	2
3670 – 4190	165	2
4820 – 5620	175	2
6670 – 8250	185	2
8250 – 8710	250	3
9860 – 11300	260	3
13000 – 15100	270	3
17900 – 21900	280	3
21900 – 25200	350	4
28800 – 33200	360	4
38700 – 45700	370	4
56200 – 62800	445	5
71800 – 82600	455	5
96200 – 113000	465	5
136000 – 143000	475	5
143000 – 154000	540	6
176000 – 200000	550	6

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.20$ %

$p_2 = 3.50$ % (left edge of column cut off in scan)

N	n	c Accept
– 252		
– 348	5	0
– 740	15	0
– 840	60	1
– 1110	70	1
– 1560	80	1
– 2380	90	1
– 3230	150	2
– 4400	160	2
– 6360	170	2
– 8400	180	2
– 9460	235	3
– 12900	245	3
– 18600	255	3
– 24400	265	3
– 26400	320	4
– 36100	330	4
– 51800	340	4
– 69100	350	4
– 72000	405	5
– 98200	415	5
– 140000	425	5
– 192000	435	5
– 200000	495	6

$p_2 = 4.00$ %

N	n	c Accept
1 – 156		
157 – 218	5	0
309 – 447	15	0
640 – 730	60	1
857 – 1020	70	1
1230 – 1510	80	1
1900 – 2580	90	1
2580 – 2900	140	2
3420 – 4100	150	2
4980 – 6170	160	2
7860 – 8710	170	2
8710 – 10100	220	3
12000 – 14500	230	3
17800 – 22300	240	3
27600 – 33800	300	4
40500 – 49200	310	4
60800 – 76900	320	4
84500 – 92700	375	5
110000 – 133000	385	5
163000 – 200000	395	5

$p_2 = 5.00$ %

N	n	c Accept
1 – 79		
80 – 118	5	0
174 – 256	15	0
393 – 533	25	0
534 – 666	60	1
825 – 1040	70	1
1340 – 1780	80	1
2470 – 2700	125	2
3330 – 4180	135	2
5340 – 7040	145	2
9580 – 11900	195	3
14900 – 19000	205	3
24900 – 34800	215	3
34800 – 40900	260	4
51000 – 64700	270	4
84000 – 113000	280	4
124000 – 137000	325	5
170000 – 200000	335	5

$p_2 = 6.00$ %

N	n	c Accept
1 – 48		
49 – 78	5	0
123 – 188	15	0
299 – 468	25	0
469 – 554	55	1
716 – 941	65	1
1270 – 1800	75	1
2450 – 2950	115	2
3840 – 5100	125	2
7010 – 10600	135	2
10600 – 14100	175	3
18600 – 25000	185	3
35000 – 44000	195	3
44000 – 49900	230	4
65000 – 86400	240	4
118000 – 175000	250	4
175000 – 200000	290	5

$p_2 = 7.00$ %

N	n	c Accept
1 – 32		
33 – 58	5	0
97 – 156	15	0
259 – 428	25	0
429 – 465	50	1
625 – 857	60	1
1220 – 1820	70	1
2470 – 3020	105	2
4110 – 5780	115	2
8470 – 11900	125	2
11900 – 13200	155	3
17900 – 24900	165	3
36000 – 54600	175	3
54600 – 74700	210	4
103000 – 147000	220	4

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.25$ %

$p_2 = 1.50$ % (left edge cut off)

N	n	c
	Accept	
– 17100	250	3
– 18600	375	4
– 21400	385	4
– 24700	395	4
– 29200	515	5
– 31300	525	5
– 36200	535	5
– 42700	655	6
– 46100	665	6
– 53200	675	6
– 62700	800	7
– 72700	810	7
– 84500	820	7
– 100000	940	8
– 107000	950	8
– 124000	960	8
– 146000	1085	9
– 168000	1095	9
– 196000		

$p_2 = 1.75$ %

N	n	c
1 – 7520	Accept	
7520 – 7670	140	2
8380 – 9380	150	2
9380 – 9550	250	3
10200 – 11100	260	3
12000 – 13200	270	3
14600 – 15300	280	3
15300 – 16200	380	4
17500 – 19100	390	4
20900 – 23100	400	4
25500 – 27600	510	5
30000 – 32700	520	5
36000 – 39900	530	5
42400 – 43100	635	6
46700 – 50800	645	6
55700 – 61400	655	6
68300 – 70300	665	6
70300 – 72400	765	7
78600 – 85700	775	7
94000 – 104000	785	7
116000 – 121000	895	8
132000 – 144000	905	8
158000 – 175000	915	8
191000 – 200000	1025	9

$p_2 = 2.00$ %

N	n	c
1 – 4100	Accept	
4100 – 4290	55	1
4290 – 4480	140	2
4840 – 5280	150	2
5820 – 6500	160	2
7490 – 7920	255	3
8620 – 9450	265	3
10400 – 11700	275	3
13200 – 13700	285	3
13700 – 15000	375	4
16500 – 18200	385	4
20200 – 22700	395	4
24700 – 25700	490	5
28100 – 30900	500	5
34300 – 38400	510	5
44200 – 47400	610	6
52000 – 57500	620	6
64000 – 72000	630	6
78800 – 86800	730	7
95600 – 106000	740	7
119000 – 134000	750	7
139000 – 144000	845	8
158000 – 175000	855	8
194000 – 200000	865	8

$p_2 = 2.50$ %

N	n	c
1 – 1510	Accept	
1510 – 1610	55	1
1800 – 2040	65	1
2370 – 2830	75	1
2830 – 3040	145	2
3370 – 3760	155	2
4250 – 4860	165	2
5660 – 6200	175	2
6200 – 6420	245	3
7130 – 7980	255	3
9010 – 10300	265	3
12000 – 13000	275	3
13000 – 14400	350	4
16100 – 18200	360	4
20700 – 24000	370	4
26700 – 28300	450	5
31600 – 35600	460	5
40600 – 46800	470	5
54100 – 61100	555	6
68800 – 78200	565	6
89800 – 104000	575	6
108000 – 117000	655	7
132000 – 149000	665	7
170000 – 200000	675	7

$p_2 = 3.00$ %

N	n	c
1 – 586	Accept	
587 – 863	5	0
864 – 884	50	1
983 – 1110	60	1
1260 – 1470	70	1
1740 – 2130	80	1
2350 – 2430	140	2
2730 – 3110	150	2
3590 – 4190	160	2
5000 – 5840	170	2
5840 – 6710	235	3
7680 – 8880	245	3
10400 – 12500	255	3
13700 – 15700	325	4
18000 – 20900	335	4
24500 – 29400	345	4
31000 – 35600	415	5
41000 – 47600	425	5
56200 – 69200	435	5
69200 – 79800	505	6
91900 – 107000	515	6
126000 – 153000	525	6
153000 – 177000	595	7

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.25$ %

$p_2 = 3.50$ %

N	n	c
1 – 259	Accept	
60 – 370	5	0
54 – 657	15	0
58 – 736	55	1
46 – 984	65	1
70 – 1410	75	1
50 – 2120	85	1
20 – 2210	135	2
50 – 2980	145	2
30 – 4260	155	2
70 – 5790	165	2
90 – 6690	220	3
20 – 9270	230	3
00 – 13800	240	3
00 – 16600	300	4
00 – 23100	310	4
00 – 34300	320	4
00 – 40300	380	5
00 – 55900	390	5
00 – 82800	400	5
00 – 96200	460	6
00 – 133000	470	6
00 – 200000	480	6

$p_2 = 4.00$ %

N	n	c
1 – 154	Accept	
155 – 220	5	0
320 – 486	15	0
556 – 618	55	1
728 – 866	65	1
1050 – 1310	75	1
1680 – 2000	85	1
2000 – 2160	130	2
2560 – 3070	140	2
3750 – 4700	150	2
5880 – 6570	205	3
7830 – 9480	215	3
11700 – 14800	225	3
16300 – 19100	280	4
22900 – 27900	290	4
34800 – 43700	300	4
43700 – 45500	350	5
54200 – 65500	360	5
80400 – 101000	370	5
116000 – 127000	425	6
152000 – 184000	435	6

$p_2 = 5.00$ %

N	n	c
1 – 75	Accept	
76 – 114	5	0
171 – 258	15	0
414 – 453	25	0
454 – 533	55	1
660 – 831	65	1
1080 – 1440	75	1
1870 – 2230	120	2
2780 – 3550	130	2
4660 – 6290	140	2
6290 – 6640	180	3
8230 – 10400	190	3
13500 – 18100	200	3
20000 – 23300	245	4
29200 – 37400	255	4
49400 – 62100	265	4
62100 – 80400	310	5
102000 – 132000	320	5
186000 – 200000	370	6

$p_2 = 6.00$ %

N	n	c
1 – 45	Accept	
46 – 74	5	0
119 – 185	15	0
304 – 404	25	0
405 – 542	55	1
712 – 960	65	1
1360 – 1830	75	1
1830 – 2310	110	2
3030 – 4100	120	2
5800 – 6920	130	2
6920 – 8730	165	3
11500 – 15700	175	3
22300 – 24600	185	3
24600 – 31700	220	4
42000 – 57300	230	4
84900 – 112000	275	5
149000 – 200000	285	5

$p_2 = 7.00$ %

N	n	c
1 – 30	Accept	
31 – 55	5	0
93 – 151	15	0
260 – 365	25	0
366 – 451	50	1
615 – 864	60	1
1280 – 1820	70	1
1820 – 2230	100	2
3060 – 4360	110	2
6570 – 7600	120	2
7600 – 9960	150	3
13800 – 19900	160	3
30000 – 42800	200	4
60000 – 87800	210	4
115000 – 131000	245	5
180000 – 200000	255	5

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.30$ %

= 1.50 %

N	n	c Accept
- 18700		
- 19500	335	4
- 22900	345	4
- 24000	460	5
- 27700	470	5
- 32700	480	5
- 34400	595	6
- 40100	605	6
- 45900	615	6
- 49600	730	7
- 58300	740	7
- 66500	860	8
- 77600	870	8
- 91200	880	8
- 95800	995	9
- 112000	1005	9
- 129000	1015	9
- 138000	1130	10
- 163000	1140	10
- 200000	1265	11

p_1 = 1.75 %

N	n	c Accept
1 - 8220		
8220 - 8270	125	2
8360 - 8630	225	3
9310 - 10100	235	3
11100 - 12200	245	3
12200 - 13000	345	4
14100 - 15400	355	4
17000 - 18600	365	4
18600 - 19700	465	5
21500 - 23500	475	5
26000 - 28500	485	5
28500 - 29800	585	6
32400 - 35500	595	6
39200 - 43500	605	6
43500 - 44700	705	7
48700 - 53300	715	7
58800 - 66300	725	7
66300 - 72700	830	8
79500 - 87700	840	8
97600 - 101000	850	8
101000 - 108000	950	9
118000 - 130000	960	9
145000 - 153000	970	9
153000 - 160000	1070	10
175000 - 193000	1080	10

p_1 = 2.00 %

N	n	c Accept
1 - 4270		
4270 - 4510	130	2
4940 - 5500	140	2
6260 - 6920	235	3
7590 - 8400	245	3
9420 - 10400	255	3
10400 - 11500	345	4
12600 - 14000	355	4
15700 - 17200	365	4
17200 - 18700	455	5
20600 - 22900	465	5
25700 - 28300	475	5
28300 - 30200	565	6
33300 - 36900	575	6
41400 - 46300	585	6
46300 - 48200	675	7
53100 - 59000	685	7
66000 - 75300	695	7
75300 - 84300	790	8
93500 - 104000	800	8
118000 - 122000	810	8
122000 - 133000	900	9
147000 - 164000	910	9
185000 - 196000	920	9
196000 - 200000	1010	10

p_1 = 2.50 %

N	n	c Accept
1 - 1520		
1520 - 1570	50	1
1760 - 2040	60	1
2450 - 2710	135	2
3020 - 3410	145	2
3900 - 4540	155	2
4880 - 5340	230	3
5970 - 6760	240	3
7750 - 9040	250	3
9360 - 10000	325	4
11300 - 12700	335	4
14600 - 17000	345	4
17500 - 18400	420	5
20600 - 23400	430	5
26800 - 31200	440	5
32400 - 37200	520	6
42100 - 48200	530	6
56000 - 59200	540	6
59200 - 66300	615	7
75000 - 85800	625	7
99400 - 107000	635	7
107000 - 117000	710	8
133000 - 151000	720	8
175000 - 193000	730	8
193000 - 200000	805	9

p_1 = 3.00 %

N	n	c Accept
1 - 675		
676 - 812	5	0
813 - 889	50	1
1000 - 1150	60	1
1350 - 1620	70	1
1970 - 2260	135	2
2580 - 2990	145	2
3530 - 4260	155	2
4450 - 5070	220	3
5810 - 6770	230	3
8010 - 9470	240	3
9470 - 10800	305	4
12400 - 14500	315	4
17200 - 19600	325	4
19600 - 22300	390	5
25700 - 30100	400	5
35800 - 39700	410	5
39700 - 45300	475	6
52400 - 61400	485	6
73300 - 79[illegible]	495	6
79800 - 91200	560	7
106000 - 124000	570	7
148000 - 159000	580	7
159000 - 182000	645	8

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.30\ \%$

$p_2 = 3.50\ \%$

N	n	c
1 – 262	Accept	
3 – 386	5	0
0 – 638	50	1
33 – 853	60	1
10 – 1230	70	1
50 – 1750	80	1
50 – 1980	130	2
10 – 2740	140	2
10 – 4120	150	2
20 – 4620	205	3
90 – 6380	215	3
90 – 9520	225	3
00 – 11900	285	4
00 – 16900	295	4
00 – 22500	305	4
00 – 25500	360	5
00 – 36100	370	5
00 – 49700	380	5
00 – 54000	435	6
00 – 75900	445	6
00 – 108000	455	6
00 – 113000	510	7
00 – 158000	520	7
00 – 200000	530	7

$p_2 = 4.00\ \%$

N	n	c
1 – 149	Accept	
150 – 217	5	0
326 – 492	15	0
493 – 519	50	1
609 – 724	60	1
881 – 1100	70	1
1420 – 1620	80	1
1620 – 1870	125	2
2240 – 2720	135	2
3400 – 4320	145	2
4320 – 4970	195	3
5960 – 7290	205	3
9160 – 10800	215	3
10800 – 12500	265	4
15100 – 18500	275	4
23400 – 26200	285	4
26200 – 30700	335	5
37200 – 45900	345	5
58100 – 62400	355	5
62400 – 74400	405	6
90200 – 112000	415	6
146000 – 178000	475	7

$p_2 = 5.00\ \%$

N	n	c
1 – 70	Accept	
71 – 109	5	0
166 – 256	15	0
398 – 423	50	1
521 – 653	60	1
842 – 1120	70	1
1500 – 1830	115	2
2290 – 2960	125	2
3970 – 4550	135	2
4550 – 5570	175	3
7040 – 9150	185	3
12900 – 16200	235	4
20600 – 27000	245	4
35700 – 46200	295	5
59000 – 77700	305	5
96700 – 103000	350	6
130000 – 166000	360	6

$p_2 = 6.00\ \%$

N	n	c
1 – 41	Accept	
42 – 70	5	0
113 – 179	15	0
303 – 348	25	0
349 – 408	50	1
530 – 707	60	1
986 – 1440	70	1
1440 – 1790	105	2
2350 – 3210	115	2
4820 – 5320	155	3
6960 – 9380	165	3
13200 – 15500	175	3
15500 – 19800	210	4
26400 – 36500	220	4
47900 – 55000	260	5
72600 – 98900	270	5
145000 – 200000	315	6

$p_2 = 7.00\ \%$

N	n
1 – 27	Acc
28 – 51	5
88 – 145	15
256 – 321	25
322 – 436	50
605 – 874	60
1410 – 1640	95
2240 – 3190	105
4850 – 5320	115
5320 – 7480	145
10600 – 15700	155
18600 – 24000	190
33500 – 49200	200
63400 – 75300	235
104000 – 151000	245

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.35\ \%$

= 1.50 %

N	n Accept	c
- 20000		
- 20400	305	4
- 21900	420	5
- 26200	430	5
- 28300	545	6
- 33500	555	6
- 36900	670	7
- 43800	680	7
- 48300	795	8
- 57200	805	8
- 63200	920	9
- 74800	930	9
- 82600	1045	10
- 97600	1055	10
- 117000	1175	11
- 141000	1185	11
- 152000	1300	12
- 182000	1310	12
- 200000	1425	13

$p_2 = 1.75\ \%$

N	n Accept	c
1 - 8500		
8500 - 8990	210	3
9880 - 10500	220	3
10500 - 11000	315	4
11900 - 13100	325	4
14800 - 15900	430	5
17400 - 19200	440	5
21100 - 23200	545	6
25400 - 28100	555	6
30300 - 33600	660	7
36900 - 41000	670	7
43200 - 44300	770	8
48500 - 53400	780	8
59400 - 61600	790	8
61600 - 63700	885	9
69700 - 76900	895	9
85700 - 87700	905	9
87700 - 91200	1000	10
100000 - 110000	1010	10
124000 - 130000	1115	11
143000 - 158000	1125	11
176000 - 186000	1230	12

$p_2 = 2.00\ \%$

N	n Accept	c
1 - 4350		
4350 - 4510	120	2
5020 - 5480	130	2
5480 - 5900	215	3
6480 - 7200	225	3
8110 - 8430	235	3
8430 - 9270	320	4
10200 - 11400	330	4
13000 - 14400	425	5
16000 - 17900	435	5
20000 - 22100	530	6
24500 - 27500	540	6
30600 - 33700	635	7
37400 - 41900	645	7
46600 - 50900	740	8
56600 - 63500	750	8
70500 - 76600	845	9
85100 - 95600	855	9
106000 - 115000	950	10
128000 - 143000	960	10
160000 - 171000	1055	11
190000 - 200000	1065	11

$p_2 = 2.50\ \%$

N	n Accept	c
1 - 1520		
1520 - 1700	50	1
1980 - 2180	60	1
2180 - 2370	125	2
2650 - 3000	135	2
3460 - 4020	145	2
4020 - 4340	215	3
4870 - 5530	225	3
6390 - 7190	235	3
7190 - 7590	305	4
8520 - 9690	315	4
11200 - 12600	325	4
12600 - 12900	395	5
14500 - 16500	405	5
19000 - 21600	415	5
21600 - 24200	490	6
27500 - 31700	500	6
36600 - 40100	580	7
45500 - 52200	590	7
61700 - 65800	670	8
74500 - 85400	680	8
99300 - 104000	690	8
104000 - 107000	760	9
121000 - 139000	770	9
161000 - 173000	780	9
173000 - 200000	855	10

$p_2 = 3.00\ \%$

N	n Acc
1 - 761	
762 - 794	45
893 - 1020	55
1200 - 1460	65
1700 - 1830	125
2090 - 2410	135
2850 - 3440	145
3560 - 3730	205
4270 - 4940	215
5830 - 7050	225
7050 - 8230	290
9550 - 11300	30
13500 - 15400	37
17900 - 21100	38
25400 - 28400	45
32900 - 38800	46
47100 - 51600	53
59800 - 70400	54
86600 - 93100	61
108000 - 127000	62
152000 - 159000	63
159000 - 167000	69
193000 - 200000	70

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.35$ %

= 3.50 %

N	n	c
– 259	Accept	
– 394	5	0
– 624	50	1
– 861	60	1
– 1320	70	1
– 1750	125	2
– 2470	135	2
– 3400	145	2
– 3630	195	3
– 5060	205	3
– 7300	215	3
– 8290	270	4
– 11800	280	4
– 15700	340	5
– 22100	350	5
– 30800	360	5
– 34300	415	6
– 49200	425	6
– 74500	490	7
– 109000	500	7
– 136000	560	8
– 200000	570	8

$p_2 = 4.00$ %

N	n	c
1 – 142	Accept	
143 – 211	5	0
324 – 446	15	0
447 – 502	50	1
596 – 720	60	1
895 – 1150	70	1
1360 – 1600	120	2
1920 – 2370	130	2
3010 – 3360	140	2
3360 – 3690	185	3
4420 – 5420	195	3
6830 – 7770	205	3
7770 – 9610	255	4
11700 – 14700	265	4
17300 – 20300	320	5
24700 – 30800	330	5
37600 – 42200	385	6
51200 – 63500	395	6
81200 – 87000	450	7
105000 – 129000	460	7
164000 – 175000	470	7
175000 – 200000	520	8

$p_2 = 5.00$ %

N	n	c
1 – 65	Accept	
66 – 102	5	0
158 – 248	15	0
357 – 406	50	1
506 – 644	60	1
851 – 1230	70	1
1230 – 1470	110	2
1850 – 2400	120	2
3260 – 3450	130	2
3450 – 3710	165	3
4640 – 5970	175	3
7980 – 9010	185	3
9010 – 11100	225	4
14100 – 18700	235	4
22700 – 25800	280	5
32700 – 42700	290	5
56200 – 59300	335	6
74800 – 96600	345	6
130000 – 138000	355	6
138000 – 169000	395	7

$p_2 = 6.00$ %

N	n	c
1 – 37	Accept	
38 – 65	5	0
106 – 171	15	0
308 – 391	50	1
516 – 702	60	1
1010 – 1190	70	1
1190 – 1360	100	2
1790 – 2440	110	2
3620 – 4150	150	3
5480 – 7500	160	3
10500 – 12100	200	4
16000 – 22000	210	4
29800 – 34500	250	5
45800 – 63300	260	5
82500 – 96300	300	6
129000 – 179000	310	6

$p_2 = 7.00$ %

N	n
1 – 24	Acce
25 – 47	5
82 – 137	15
248 – 281	25
282 – 310	45
421 – 591	55
884 – 1170	65
1170 – 1620	95
2280 – 3430	105
3950 – 5540	140
7880 – 12500	150
12500 – 18200	185
26200 – 38500	195
38500 – 42400	225
58700 – 84800	235
118000 – 134000	270
186000 – 200000	280

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.40$ %

p_2 = 1.50 %		
N	n	c
100	Accept	
500	390	5
200	500	6
800	510	6
500	620	7
800	630	7
800	735	8
400	745	8
500	855	9
100	865	9
900	975	10
100	985	10
600	1095	11
300	1105	11
700	1215	12
13000	1225	12
19000	1335	13
44000	1345	13
51000	1455	14
82000	1465	14
92000	1575	15

p_2 = 1.75 %		
N	n	c
1 – 8820	Accept	
8820 – 9320	195	3
9320 – 9710	290	4
10600 – 11700	300	4
12400 – 13500	400	5
14800 – 16700	410	5
16700 – 17200	505	6
18800 – 20800	515	6
22700 – 23900	615	7
26200 – 29100	625	7
30800 – 33100	725	8
36500 – 40600	735	8
41800 – 45800	835	9
50600 – 56500	845	9
56500 – 63200	945	10
70000 – 76300	955	10
76300 – 79100	1050	11
87000 – 96700	1060	11
103000 – 108000	1160	12
120000 – 133000	1170	12
138000 – 149000	1270	13
164000 – 186000	1280	13
186000 – 200000	1380	14

p_2 = 2.00 %		
N	n	c
1 – 4450	Accept	
4450 – 4940	115	2
4940 – 5360	200	3
5920 – 6630	210	3
7130 – 7970	300	4
8850 – 9970	310	4
10400 – 10600	395	5
11700 – 13100	405	5
14800 – 15200	415	5
15200 – 17100	500	6
19100 – 22000	510	6
22000 – 24700	600	7
27600 – 31700	610	7
31700 – 35400	700	8
39600 – 45400	710	8
45400 – 50500	800	9
56500 – 64900	810	9
64900 – 71700	900	10
80300 – 92500	910	10
92500 – 102000	1000	11
114000 – 131000	1010	11
131000 – 143000	1100	12
160000 – 181000	1110	12
187000 – 200000	1200	13

p_2 = 2.50 %		
N	n	c
1 – 1510	Accept	
1510 – 1590	45	1
1860 – 1980	55	1
1980 – 2040	115	2
2270 – 2560	125	2
2950 – 3420	135	2
3420 – 3850	205	3
4350 – 5010	215	3
5780 – 6230	290	4
7040 – 8070	300	4
9520 – 9810	375	5
11100 – 12600	385	5
14700 – 15500	395	5
15500 – 17100	465	6
19400 – 22500	475	6
24800 – 26100	550	7
29600 – 34000	560	7
39400 – 44700	640	8
51200 – 59500	650	8
62200 – 67100	725	9
76600 – 88500	735	9
98000 – 114000	815	10
131000 – 153000	825	10
153000 – 170000	900	11
194000 – 200000	910	11

p_2 = 3.00 %		
N	n	c
1 – 722	Accept	
723 – 781	45	1
892 – 1040	55	1
1260 – 1490	65	1
1490 – 1650	120	2
1890 – 2210	130	2
2650 – 2960	140	2
2960 – 3060	195	3
3510 – 4080	205	3
4840 – 5510	215	3
5510 – 6110	275	4
7070 – 8350	285	4
9910 – 10300	350	5
11900 – 14000	360	5
16700 – 17500	370	5
17500 – 19700	430	6
23000 – 27300	440	6
30600 – 32200	505	7
37400 – 44100	515	7
52800 – 60500	585	8
70900 – 84700	595	8
90900 – 97500	660	9
114000 – 134000	670	9
156000 – 181000	740	10

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.40$ %

$p_2 = 3.50$ %

N	n	c
1 – 249	Accept	
250 – 389	5	0
497 – 526	45	1
608 – 714	55	1
864 – 1080	65	1
1290 – 1520	120	2
1800 – 2180	130	2
2770 – 3270	190	3
3890 – 4730	200	3
5600 – 6610	260	4
7880 – 9640	270	4
10900 – 13000	330	5
15500 – 19100	340	5
20700 – 25000	400	6
30000 – 37000	410	6
39000 – 47600	470	7
57400 – 72300	480	7
72300 – 75900	535	8
90000 – 109000	545	8
134000 – 142000	605	9
169000 – 200000	615	9

$p_2 = 4.00$ %

N	n	c
1 – 133	Accept	
134 – 199	5	0
312 – 406	15	0
407 – 482	50	1
579 – 710	60	1
903 – 1170	70	1
1170 – 1350	115	2
1620 – 2000	125	2
2560 – 2720	135	2
2720 – 3200	180	3
3890 – 4860	190	3
5860 – 7200	245	4
8840 – 11200	255	4
12100 – 13000	305	5
15800 – 19600	315	5
24700 – 28000	370	6
34100 – 42700	380	6
50000 – 59500	435	7
73200 – 92500	445	7
100000 – 126000	500	8
156000 – 200000	510	8

$p_2 = 5.00$ %

N	n	c
1 – 59	Accept	
60 – 94	5	0
147 – 235	15	0
324 – 387	50	1
488 – 630	60	1
854 – 1050	70	1
1050 – 1170	105	2
1460 – 1890	115	2
2570 – 2730	125	2
2730 – 3020	160	3
3600 – 4950	170	3
6600 – 7400	215	4
9390 – 12300	225	4
15500 – 17700	270	5
22600 – 29800	280	5
35800 – 41700	325	6
53500 – 71000	335	6
81500 – 97500	380	7
125000 – 167000	390	7
184000 – 200000	435	8

$p_2 = 6.00$ %

N	n	c
1 – 33	Accept	
34 – 59	5	0
98 – 159	15	0
281 – 373	50	1
497 – 691	60	1
1000 – 1340	100	2
1800 – 2570	110	2
2860 – 3190	145	3
4220 – 5830	155	3
7680 – 9530	195	4
12900 – 18200	205	4
20100 – 28000	245	5
38600 – 51000	255	5
51000 – 60900	290	6
81900 – 115000	300	6
130000 – 174000	340	7

$p_2 = 7.00$ %

N	n	c
1 – 21	Accept	
22 – 43	5	0
76 – 128	15	0
249 – 293	45	1
402 – 575	55	1
889 – 972	65	1
973 – 1150	90	2
1600 – 2350	100	2
3050 – 4030	135	3
5730 – 8830	145	3
8830 – 9820	175	4
13600 – 20000	185	4
25500 – 32200	220	5
45700 – 71500	230	5
71500 – 105000	265	6
153000 – 200000	275	6

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.50$ %

$p_2 = 1.50$ %

N	n	c
1 - 22700	Accept	
22700 - 23300	535	7
24700 - 27400	645	8
28700 - 29700	750	9
32500 - 33700	760	9
33700 - 35300	860	10
35700 - 39700	870	10
39700 - 42000	970	11
46800 - 50000	1080	12
55300 - 59600	1190	13
65400 - 70900	1300	14
77300 - 81500	1410	15
91400 - 101000	1520	16
108000 - 120000	1630	17
128000 - 142000	1740	18
151000 - 154000	1845	19
169000 - 175000	1855	19
178000 - 183000	1955	20

$p_2 = 1.75$ %

N	n	c
1 - 9030	Accept	
9030 - 9630	260	4
9630 - 10300	350	5
11300 - 11900	360	5
11900 - 13000	450	6
11400 - 11900	460	6
11900 - 16400	550	7
18200 - 18700	560	7
18700 - 20700	650	8
23500 - 26100	750	9
29500 - 32800	850	10
36900 - 41200	950	11
46200 - 51500	1050	12
57600 - 64400	1150	13
71900 - 80200	1250	14
89500 - 99800	1350	15
111000 - 124000	1450	16
138000 - 154000	1550	17
172000 - 191000	1650	18

$p_2 = 2.00$ %

N	n	c
1 - 4420	Accept	
4420 - 4950	180	3
5500 - 5830	265	4
6490 - 7370	275	4
7370 - 7610	355	5
8450 - 9530	365	5
9900 - 10900	450	6
12200 - 13200	460	6
13200 - 14000	540	7
15600 - 17600	550	7
17600 - 19800	635	8
22300 - 23300	645	8
23300 - 25000	725	9
25100 - 30000	735	9
30300 - 35200	820	10
40400 - 43900	910	11
49500 - 53000	920	11
53000 - 54900	1000	12
61500 - 69400	1010	12
69400 - 76400	1095	13
86400 - 90600	1105	13
90600 - 94900	1185	14
107000 - 116000	1195	14
118000 - 132000	1280	15
149000 - 154000	1290	15
154000 - 163000	1370	16
184000 - 200000	1380	16

$p_2 = 2.50$ %

N	n	c
1 - 1410	Accept	
1410 - 1460	40	1
1650 - 1720	105	2
1930 - 2210	115	2
2610 - 2780	185	3
3150 - 3640	195	3
4060 - 4310	265	4
4900 - 5670	275	4
6150 - 6490	345	5
7390 - 8570	355	5
9140 - 9580	425	6
10900 - 12700	435	6
13400 - 14000	505	7
15900 - 18500	515	7
19500 - 20100	585	8
23000 - 26800	595	8
28200 - 33000	670	9
38400 - 40500	680	9
40500 - 47200	750	10
54800 - 58000	760	10
58000 - 67000	830	11
77900 - 82700	840	11
82700 - 95000	910	12
110000 - 118000	920	12
118000 - 134000	990	13
156000 - 167000	1000	13
167000 - 189000	1070	14

$p_2 = 3.00$ %

N	n	c
1 - 625	Accept	
626 - 725	45	1
851 - 1030	55	1
1190 - 1260	110	2
1150 - 1700	120	2
2060 - 2170	130	2
2170 - 2510	185	3
2940 - 3540	195	3
3690 - 4080	255	4
4710 - 5690	265	4
6060 - 6400	325	5
7450 - 8860	335	5
9750 - 11400	400	6
13500 - 15400	410	6
15400 - 17300	470	7
20400 - 24100	480	7
24100 - 25900	540	8
30400 - 37600	550	8
37600 - 45200	615	9
54000 - 58400	625	9
58400 - 66900	685	10
79400 - 90000	695	10
90000 - 98300	755	11
117000 - 138000	765	11
138000 - 145000	825	12
171000 - 200000	835	12

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for p_1 = 0.50 %

p_2 = 3.50 %

N	n	c
1 - 207	Accept	
208 - 330	5	0
417 - 467	45	1
550 - 662	55	1
829 - 1000	65	1
1000 - 1080	110	2
1270 - 1540	120	2
1970 - 2120	175	3
2500 - 3040	185	3
3620 - 3870	240	4
4600 - 5610	250	4
6370 - 6850	305	5
8100 - 10000	315	5
10900 - 11900	370	6
14300 - 17600	380	6
18500 - 20400	435	7
24600 - 31000	445	7
31000 - 34800	500	8
42000 - 51800	510	8
51800 - 58900	565	9
71400 - 86200	575	9
86200 - 99300	630	10
121000 - 143000	640	10
143000 - 167000	695	11

p_2 = 4.00 %

N	n	c
1 - 106	Accept	
107 - 163	5	0
259 - 337	15	0
338 - 359	45	1
429 - 523	55	1
662 - 899	65	1
900 - 1080	110	2
1310 - 1660	120	2
1910 - 2280	170	3
2800 - 3680	180	3
3680 - 4530	230	4
4620 - 6920	240	4
6920 - 8730	290	5
10900 - 12600	300	5
12600 - 13500	345	6
16600 - 20900	355	6
22700 - 25200	405	7
31100 - 40500	415	7
40500 - 46800	465	8
58000 - 72200	475	8
72200 - 86300	525	9
108000 - 128000	535	9
128000 - 159000	585	10

p_2 = 5.00 %

N	n	c
1 - 46	Accept	
47 - 75	5	0
120 - 193	15	0
271 - 340	50	1
436 - 578	60	1
798 - 874	100	2
1100 - 1440	110	2
1870 - 2390	155	3
3120 - 4030	165	3
4030 - 4900	205	4
6370 - 8370	215	4
8370 - 9780	255	5
12700 - 17100	265	5
17100 - 19200	305	6
24800 - 34400	315	6
34400 - 37300	355	7
48000 - 64200	365	7
69700 - 92200	410	8
123000 - 138000	420	8
138000 - 176000	460	9

p_2 = 6.00 %

N	n	c
1 - 25	Accept	
26 - 47	5	0
80 - 131	15	0
232 - 250	45	1
329 - 445	55	1
643 - 755	65	1
756 - 946	95	2
1280 - 1900	105	2
1900 - 2360	140	3
3210 - 4510	150	3
4510 - 5610	185	4
7670 - 10400	195	4
10400 - 13000	230	5
17900 - 23400	240	5
23400 - 29800	275	6
41100 - 52400	285	6
52400 - 67500	320	7
93500 - 116000	330	7
116000 - 152000	365	8

p_2 = 7.00 %

N	n	c
1 - 16	Accept	
17 - 34	5	0
62 - 105	15	0
193 - 211	25	0
212 - 255	45	1
356 - 523	55	1
737 - 1060	90	2
1560 - 2010	100	2
2010 - 2820	130	3
4130 - 5180	140	3
5180 - 7150	170	4
10500 - 12900	180	4
12900 - 17700	210	5
26000 - 31900	220	5
31900 - 43300	250	6
63600 - 77600	260	6
77600 - 105000	290	7
154000 - 187000	300	7
187000 - 200000	330	8

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for p_1 = 0.60 %

p_2 = 1.50 %

N	n Accept	c
1 - 23100		
23100 - 23800	670	9
24500 - 25900	770	10
27400 - 28200	870	11
30700 - 33800	975	12
34500 - 36900	1075	13
38800 - 40300	1175	14
43700 - 49200	1280	15
49200 - 53000	1380	16
55500 - 57900	1480	17
62500 - 70400	1585	18
70400 - 76100	1685	19
79300 - 82900	1785	20
89400 - 101000	1890	21
101000 - 109000	1990	22
113000 - 118000	2090	23
128000 - 143000	2195	24
143000 - 155000	2295	25
161000 - 168000	2395	26
182000 - 200000	2500	27

p_2 = 1.75 %

N	n Accept	c
1 - 8630		
8630 - 9290	320	5
9290 - 9710	405	6
11000 - 12100	500	7
13000 - 13700	590	8
15500 - 17100	685	9
19500 - 19200	775	10
21400 - 22000	785	10
22000 - 23900	870	11
26100 - 26700	960	12
29700 - 30900	970	12
30900 - 33100	1055	13
36600 - 41000	1150	14
[illegible]3200 - [illegible]	12[illegible]0	15
51000 - 56100	1335	16
60100 - 62100	1425	17
70800 - 75500	1520	18
[illegible]3500 - [illegible]	1615	19
[illegible]8100 - 104000	1705	20
115000 - 128000	1800	21
135000 - 1[illegible]0000	1890	22
159000 - 172000	1985	23
187000 - 1[illegible]9000	2075	24

p_2 = 2.00 %

N	n Accept	c
1 - 4010		
4010 - 4420	165	3
4420 - 4920	245	4
5590 - 6250	330	5
7100 - 7860	415	6
8980 - 9810	500	7
11300 - 12100	585	8
13700 - 14200	595	8
14200 - 15000	670	9
16900 - 17700	680	9
17700 - 18300	755	10
20600 - 22000	765	10
22000 - 25200	845	11
27300 - 30500	930	12
33700 - 37000	1015	13
41500 - 44700	1100	14
51100 - 53900	1185	15
61200 - 62800	1195	15
62800 - 65000	1270	16
73600 - 77200	1280	16
77200 - 83300	1360	17
94600 - 106000	1445	18
116000 - 127000	1530	19
142000 - 152000	1615	20
173000 - 182000	1700	21

p_2 = 2.50 %

N	n Accept	c
1 - 1190		
1190 - 1320	40	1
1360 - 1440	100	2
1630 - 1900	110	2
2050 - 2230	175	3
2550 - 3010	185	3
3010 - 3300	250	4
3790 - 4320	260	4
4320 - 4720	325	5
5440 - 6040	335	5
6040 - 6620	400	6
7650 - 8340	410	6
8340 - 9140	475	7
10600 - 11400	485	7
11400 - 12500	550	8
14500 - 15400	560	8
15400 - 16900	625	9
19700 - 20700	635	9
20700 - 22800	700	10
26600 - 27800	710	10
27800 - 30700	775	11
35800 - 37100	785	11
37100 - 41000	850	12
47900 - 49400	860	12
49400 - 54800	925	13
65500 - 72900	1000	14
86900 - 96800	1075	15
115000 - 128000	1150	16
152000 - 170000	1225	17

p_2 = 3.00 %

N	n Accept	c
1 - 493		
494 - 524	40	1
606 - 720	50	1
895 - 952	60	1
953 - 987	105	2
1140 - 1350	115	2
1650 - 1840	175	3
2160 - 2640	185	3
2640 - 2760	240	4
3220 - 3860	250	4
4080 - 4650	310	5
5520 - 6120	320	5
6120 - 6560	375	6
7730 - 9040	385	6
9040 - 10700	445	7
13100 - 14700	510	8
17500 - 19100	520	8
19100 - 20000	575	9
23600 - 27500	585	9
27500 - 31900	645	10
39300 - 43100	710	11
51300 - 56600	720	11
56600 - 68600	780	12
80400 - 91600	845	13
110000 - 114000	855	13
114000 - 122000	910	14
136000 - 163000	920	14
163000 - 193000	980	15

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.60\ \%$

$p_2 = 3.50\ \%$

N	n	c
1 – 144	Accept	
145 – 225	5	0
340 – 382	45	1
454 – 552	55	1
702 – 800	65	1
801 – 960	110	2
1160 – 1480	120	2
1480 – 1700	170	3
2050 – 2540	180	3
2540 – 2610	230	4
3380 – 4150	240	4
4150 – 4470	290	5
5370 – 6610	300	5
6610 – 6910	350	6
8350 – 10400	360	6
10400 – 12600	415	7
16100 – 19400	475	8
24300 – 29300	535	9
26000 – 38100	545	9
38100 – 43900	595	10
53900 – 58200	605	10
58200 – 65600	655	11
80400 – 88700	665	11
88700 – 97700	715	12
119000 – 135000	725	12
135000 – 145000	775	13
177000 – 200000	785	13

$p_2 = 4.00\ \%$

N	n	c
1 – 74	Accept	
75 – 115	5	0
181 – 279	15	0
280 – 292	45	1
352 – 432	55	1
553 – 714	65	1
715 – 786	105	2
957 – 1200	115	2
1420 – 1780	165	3
2230 – 2550	175	3
2550 – 3090	220	4
3870 – 4390	230	4
4390 – 5170	275	5
6470 – 7330	285	5
7390 – 8490	330	6
10600 – 12300	340	6
12300 – 13800	385	7
17200 – 20200	395	7
20200 – 22100	440	8
27600 – 33000	450	8
33000 – 35400	495	9
44000 – 53600	505	9
53600 – 56300	550	10
69800 – 87000	560	10
87000 – 110000	610	11
140000 – 174000	665	12

$p_2 = 5.00\ \%$

N	n	c
1 – 31	Accept	
32 – 54	5	0
88 – 139	15	0
227 – 280	50	1
361 – 483	60	1
633 – 759	100	2
981 – 1360	110	2
1360 – 1740	150	3
2300 – 2690	160	3
2690 – 2950	195	4
3810 – 5100	205	4
5100 – 6200	245	5
8160 – 9620	255	5
9620 – 12900	295	6
17500 – 20400	340	7
26900 – 32300	350	7
32300 – 41800	390	8
58000 – 65400	435	9
85700 – 106000	445	9
106000 – 132000	485	10
177000 – 190000	495	10
190000 – 200000	530	11

$p_2 = 6.00\ \%$

N	n	c
1 – 16	Accept	
17 – 34	5	0
60 – 97	15	0
167 – 202	25	0
203 – 271	50	1
371 – 541	60	1
597 – 825	95	2
1150 – 1370	105	2
1370 – 1620	135	3
2200 – 2960	145	3
2960 – 4040	180	4
5730 – 6120	190	4
6120 – 7290	220	5
10100 – 12700	230	5
12700 – 17700	265	6
25200 – 31000	305	7
43200 – 51500	315	7
51500 – 71000	350	8
101000 – 128000	390	9
179000 – 200000	400	9

$p_2 = 7.00\ \%$

N	n	c
1 – 9	Accept	
10 – 25	5	0
47 – 80	15	0
142 – 182	25	0
183 – 211	45	1
295 – 437	55	1
569 – 662	85	2
935 – 1400	95	2
1400 – 1820	125	3
2650 – 3320	135	3
3320 – 4840	165	4
7340 – 8830	200	5
12600 – 16600	210	5
16600 – 22600	240	6
35700 – 40500	275	7
58200 – 79100	285	7
79100 – 102000	315	8
151000 – 172000	325	8
172000 – 180000	350	9

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for $p_1 = 0.70\ \%$

$p_2 = 1.50\ \%$

N	n	c
1 – 21600	Accept	
21600 – 22400	800	11
22400 – 23500	890	12
24300 – 25500	985	13
26400 – 27700	1080	14
28800 – 30100	1175	15
31300 – 32800	1270	16
34200 – 35700	1365	17
37200 – 38800	1460	18
40600 – 42200	1555	19
44300 – 45900	1650	20
48300 – 49900	1745	21
52600 – 54200	1840	22
57300 – 58900	1935	23
62500 – 63900	2030	24
68000 – 69300	2125	25
74100 – 75200	2220	26
80700 – 87800	2320	27
87800 – 95500	2415	28
95500 – 104000	2510	29
104000 – 113000	2605	30
113000 – 123000	2700	31
123000 – 133000	2795	32
133000 – 145000	2890	33
145000 – 158000	2985	34
158000 – 171000	3080	35
171000 – 186000	3175	36
186000 – 200000	3270	37

$p_2 = 1.75\ \%$

N	n	c
1 – 7440	Accept	
7440 – 8330	380	6
8330 – 9140	465	7
9550 – 10100	550	8
11000 – 12600	640	9
12600 – 13600	725	10
14500 – 15000	810	11
16500 – 18300	900	12
18900 – 19900	985	13
21600 – 24600	1075	14
24600 – 26400	1160	15
28000 – 31800	1250	16
31800 – 34700	1335	17
36100 – 37500	1420	18
41000 – 45400	1510	19
46400 – 48900	1595	20
52600 – 59400	1685	21
59400 – 63600	1770	22
67300 – 75900	1860	23
75900 – 82500	1945	24
85700 – 88500	2030	25
96700 – 107000	2120	26
109000 – 114000	2205	27
123000 – 138000	2295	28
138000 – 147000	2380	29
156000 – 176000	2470	30
176000 – 190000	2555	31
198000 – 200000	2640	32

$p_2 = 2.00\ \%$

N	n	c
1 – 3220	Accept	
3220 – 3300	155	3
3470 – 3630	230	4
4090 – 4250	240	4
4250 – 4470	310	5
5050 – 5220	320	5
5220 – 5460	390	6
6170 – 6380	400	6
6380 – 6600	470	7
7450 – 7760	480	7
7760 – 8930	555	8
9360 – 10600	635	9
11200 – 12600	715	10
13400 – 14800	795	11
16000 – 17300	875	12
19000 – 20200	955	13
22400 – 23600	1035	14
26500 – 27400	1115	15
31300 – 36800	1200	16
36800 – 41900	1280	17
43300 – 48400	1360	18
50800 – 55800	1440	19
59500 – 64400	1520	20
69700 – 74200	1600	21
81600 – 85400	1680	22
95400 – 98200	1760	23
112000 – 130000	1845	24
130000 – 148000	1925	25
152000 – 169000	2005	26
177000 – 194000	2085	27

$p_2 = 2.50\ \%$

N	n	c
1 – 826	Accept	
827 – 902	40	1
1050 – 1120	100	2
1290 – 1520	110	2
1580 – 1680	170	3
1930 – 2260	180	3
2260 – 2360	240	4
2700 – 3130	250	4
3130 – 3660	315	5
4200 – 4820	385	6
5540 – 6260	455	7
7230 – 8020	525	8
9340 – 10200	595	9
12000 – 12800	665	10
15300 – 16100	735	11
18800 – 19500	745	11
19500 – 23400	810	12
24700 – 29200	880	13
31200 – 36200	950	14
39400 – 44800	1020	15
49500 – 55400	1090	16
62200 – 68300	1160	17
78000 – 84200	1230	18
97700 – 104000	1300	19
122000 – 127000	1370	20
153000 – 183000	1445	21
192000 – 200000	1515	22

$p_2 = 3.00\ \%$

N	n	c
1 – 220	Accept	
221 – 365	5	0
366 – 425	45	1
503 – 612	55	1
752 – 885	110	2
1050 – 1270	120	2
1270 – 1320	170	3
1550 – 1860	180	3
1950 – 2130	235	4
2520 – 2880	245	4
2880 – 3290	300	5
3950 – 4120	310	5
4120 – 5000	365	6
5750 – 6280	425	7
7500 – 7970	435	7
7970 – 9250	430	8
10900 – 11400	550	9
13600 – 14900	560	9
14900 – 16600	615	10
20200 – 24000	680	11
27200 – 29100	740	12
34900 – 36700	750	12
36700 – 41800	805	13
43400 – 60300	870	14
66100 – 72200	930	15
88300 – 103000	995	16
118000 – 124000	1055	17
149000 – 158000	1065	17
158000 – 176000	1120	18

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Single Sampling Tables for p_1 = 0.70 %

p_2 = 3.50 %

N	n	c
1 – 80	Accept	
81 – 122	5	0
190 – 272	15	0
273 – 324	50	1
393 – 489	60	1
634 – 746	110	2
907 – 1140	120	2
1140 – 1190	165	3
1420 – 1760	175	3
1850 – 2090	225	4
2540 – 2880	235	4
2880 – 3550	285	5
4310 – 4860	340	6
5970 – 6400	350	6
6400 – 7990	400	7
9320 – 10600	455	8
13500 – 14100	510	9
17200 – 19500	520	9
19500 – 22600	570	10
27900 – 29700	625	11
36400 – 40100	635	11
40100 – 47300	685	12
57100 – 61600	740	13
75900 – 81400	750	13
81400 – 97800	800	14
115000 – 127000	855	15
157000 – 164000	865	15
164000 – 200000	915	16

p_2 = 4.00 %

N	n	c
1 – 42	Accept	
43 – 68	5	0
105 – 162	15	0
233 – 256	50	1
314 – 394	60	1
520 – 572	70	1
573 – 613	105	2
748 – 945	115	2
1080 – 1210	160	3
1490 – 1830	170	3
1830 – 2220	215	4
2810 – 2960	225	4
2960 – 3210	265	5
3980 – 4700	275	5
4700 – 5570	320	6
7260 – 7750	370	7
9670 – 11300	380	7
11300 – 13200	425	8
17200 – 22600	480	9
26200 – 30500	530	10
39700 – 51700	585	11
59900 – 69200	635	12
90200 – 117000	690	13
136000 – 155000	740	14

p_2 = 5.00 %

N	n	c
1 – 17	Accept	
18 – 33	5	0
56 – 87	15	0
140 – 194	25	0
195 – 209	50	1
267 – 351	60	1
505 – 595	100	2
771 – 1020	110	2
1020 – 1120	145	3
1450 – 1900	155	3
1900 – 2570	195	4
3360 – 4420	240	5
5820 – 7450	285	6
9940 – 12400	330	7
16800 – 20600	375	8
23400 – 33900	420	9
47500 – 55700	465	10
74500 – 80000	475	10
80000 – 91000	510	11
121000 – 134000	520	11
134000 – 148000	555	12

p_2 = 6.00 %

N	n	c
1 – 8	Accept	
9 – 21	5	0
40 – 65	15	0
107 – 172	25	0
173 – 205	50	1
277 – 394	60	1
480 – 649	95	2
914 – 1030	105	2
1030 – 1330	135	3
1870 – 2040	145	3
2040 – 2570	175	4
3620 – 3910	185	4
3910 – 4840	215	5
6830 – 7390	225	5
7390 – 9000	255	6
12700 – 13800	265	6
13800 – 16600	295	7
23400 – 25600	305	7
25600 – 30300	335	8
42700 – 47100	345	8
47100 – 55200	375	9
77700 – 86500	385	9
86[illegible]0 – 100000	415	10
140000 – 158000	425	10
158000 – 181000	455	11

p_2 = 7.00 %

N	n	c
1 – 5	Reject	
6 – 16	5	0
33 – 55	15	0
95 – 160	25	0
161 – 223	50	1
326 – 455	60	1
456 – 520	85	2
738 – 1050	95	2
1050 – 1490	125	3
2220 – 2890	160	4
4560 – 5490	195	5
8080 – 9600	205	5
9600 – 15000	235	6
19300 – 27600	270	7
38400 – 50700	305	8
76000 – 93000	340	9
138000 – 152000	350	9
152000 – 170000	375	10

For N between two intervals adjacent in the table find (n,c) for the first of these intervals and use (n+5,c) as optimum plan.

Relation between p_r and w_2 for fixed $(p_{1o}, p_{2o}, \gamma_2)$.

Use the same sampling plan for $w_2 = 0.05$ and $p_{ro} = 0.01$ (0.10) as for w_2 and $p_r = 0.01f$ (0.10f) where f is given in the table.

	$\rho_2 = p_{2o}/p_{ro}$					ρ_1
w_2	1.5	2.0	3.0	5.0	7.0	$= p_{1o}/p_{ro}$
1	.51	.44	.40	.38	.37	.2
	.78	.77	.76	.76	.76	.7
2	.70	.63	.58	.55	.53	.2
	.85	.84	.83	.82	.82	.7
3	.83	.78	.73	.70	.69	.2
	.91	.89	.89	.88	.88	.7
4	.93	.90	.87	.86	.85	.2
	.96	.95	.94	.94	.94	.7
5	1.00	1.00	1.00	1.00	1.00	.2
	1.00	1.00	1.00	1.00	1.00	.7
6	1.06	1.09	1.11	1.14	1.15	.2
	1.04	1.05	1.05	1.06	1.06	.7
7	1.10	1.16	1.22	1.27	1.29	.2
	1.07	1.09	1.11	1.12	1.12	.7
8	1.14	1.22	1.31	1.39	1.43	.2
	1.10	1.13	1.16	1.17	1.18	.7
9	1.18	1.28	1.40	1.51	1.56	.2
	1.12	1.17	1.21	1.23	1.24	.7
10	1.20	1.33	1.48	1.63	1.69	.2
	1.15	1.20	1.25	1.29	1.30	.7
12	1.25	1.41	1.63	1.84	1.95	.2
	1.19	1.27	1.34	1.40	1.42	.7
14	1.28	1.48	1.75	2.03	2.19	.2
	1.22	1.33	1.43	1.51	1.54	.7
16	1.31	1.54	1.86	2.22	2.41	.2
	1.25	1.38	1.51	1.62	1.67	.7
18	1.33	1.58	1.95	2.38	2.63	.2
	1.27	1.42	1.59	1.72	1.79	.7
20	1.35	1.63	2.03	2.54	2.84	.2
	1.29	1.46	1.66	1.83	1.91	.7

Table of b_1, b_2, and b_3.

$\rho_1 = p_1/p_r$	$\rho_2 = p_2/p_r$ 1.5	2.0	3.0	5.0	7.0	
0.2	0.63	0.64	0.65	0.66	0.67	b_1
	0.24	0.27	0.31	0.35	0.37	b_2
	1.42	1.29	1.15	1.03	0.96	b_3
0.3	0.61	0.62	0.64	0.65	0.65	b_1
	0.19	0.23	0.27	0.31	0.34	b_2
	1.69	1.48	1.28	1.11	1.03	b_3
0.4	0.60	0.61	0.63	0.64	0.64	b_1
	0.16	0.19	0.24	0.29	0.32	b_2
	1.99	1.69	1.41	1.19	1.08	b_3
0.5	0.59	0.60	0.62	0.63	0.63	b_1
	0.13	0.17	0.21	0.27	0.30	b_2
	2.33	1.91	1.54	1.27	1.14	b_3
0.6	0.58	0.59	0.61	0.62	0.62	b_1
	0.11	0.15	0.19	0.25	0.28	b_2
	2.74	2.15	1.68	1.34	1.19	b_3
0.7	0.58	0.59	0.60	0.62	0.62	b_1
	0.09	0.13	0.18	0.23	0.26	b_2
	3.24	2.42	1.82	1.42	1.25	b_3

Conversion factor f_2 for N due to a change in $p_r = p_s$ for fixed (p_1,p_2,w_2).

Use $N^* = Nf_2$ as argument in the master table to find (n^*,c^*).

$p_r = p_s = 0.01\lambda$ or 0.10λ, (p_1,p_2,w_2) are given in the master tables,
$\rho_1 = 100p_1$ or $10p_1$, $\rho_2 = 100p_2$ or $10p_2$.

ρ_2	ρ_1	λ= 0.50	0.60	0.70	0.80	0.90	1.00	1.25	1.50	1.75	2.00	3.00
1.5	0.2	1.80	1.52	1.33	1.18	1.08	1.00	-	-	-	-	-
	0.3	2.21	1.79	1.50	1.28	1.12	1.00	-	-	-	-	-
	0.4	-	2.11	1.69	1.38	1.16	1.00	-	-	-	-	-
	0.5	-	-	1.90	1.50	1.21	1.00	-	-	-	-	-
	0.6	-	-	-	1.64	1.27	1.00	-	-	-	-	-
	0.7	-	-	-	-	-	1.00	-	-	-	-	-
2.0	0.2	1.69	1.47	1.30	1.18	1.08	1.00	0.87	-	-	-	-
	0.3	1.92	1.63	1.41	1.24	1.10	1.00	0.82	-	-	-	-
	0.4	-	1.79	1.52	1.30	1.13	1.00	0.78	-	-	-	-
	0.5	-	-	1.62	1.36	1.16	1.00	0.73	0.58	-	-	-
	0.6	-	-	-	1.42	1.19	1.00	0.69	0.52	-	-	-
	0.7	-	-	-	-	1.21	1.00	0.65	0.47	-	-	-
3.0	0.2	1.53	1.38	1.25	1.15	1.07	1.00	0.88	0.80	-	-	-
	0.3	1.64	1.46	1.31	1.19	1.08	1.00	0.85	0.74	0.67	-	-
	0.4	-	1.52	1.36	1.22	1.10	1.00	0.82	0.70	0.62	0.56	-
	0.5	-	-	-	1.24	1.11	1.00	0.80	0.66	0.57	0.50	-
	0.6	-	-	-	-	1.12	1.00	0.78	0.63	0.53	0.46	-
	0.7	-	-	-	-	-	1.00	0.76	0.60	0.49	0.41	-
5.0	0.2	1.39	1.28	1.19	1.12	1.05	1.00	0.89	0.82	0.76	0.72	-
	0.3	-	1.31	1.22	1.13	1.06	1.00	0.88	0.79	0.72	0.67	-
	0.4	-	-	1.23	1.14	1.07	1.00	0.86	0.76	0.69	0.63	0.50
	0.5	-	-	-	1.14	1.07	1.00	0.85	0.74	0.66	0.60	0.46
	0.6	-	-	-	-	-	1.00	0.85	0.73	0.65	0.58	0.42
	0.7	-	-	-	-	-	1.00	0.85	0.73	0.63	0.56	0.39
7.0	0.2	1.31	1.23	1.16	1.10	1.05	1.00	0.91	0.84	0.79	0.74	0.65
	0.3	-	1.24	1.17	1.11	1.05	1.00	0.89	0.81	0.75	0.70	0.59
	0.4	-	-	-	1.11	1.05	1.00	0.89	0.80	0.74	0.68	0.55
	0.5	-	-	-	-	1.05	1.00	0.89	0.79	0.72	0.66	0.52
	0.6	-	-	-	-	-	1.00	0.89	0.79	0.72	0.65	0.50
	0.7	-	-	-	-	-	1.00	0.90	0.80	0.72	0.65	0.48

<u>Correction g_2 to n^* due to a change in $p_r = p_s$.</u>

Reference value $p_r = p_s = 0.010$, $n = n^* + g_2$.

ρ_2	ρ_1	λ = 0.50	0.60	0.70	0.80	0.90	1.00	1.25	1.50	1.75	2.00	3.00
1.5	0.2	130	100	70	50	25	0	-	-	-	-	-
	0.3	160	120	85	55	30	0	-	-	-	-	-
	0.4	-	150	105	65	35	0	-	-	-	-	-
	0.5	-	-	135	85	40	0	-	-	-	-	-
	0.6	-	-	-	115	50	0	-	-	-	-	-
	0.7	-	-	-	-	-	0	-	-	-	-	-
2.0	0.2	75	55	40	25	15	0	-30	-	-	-	-
	0.3	95	70	50	30	15	0	-35	-	-	-	-
	0.4	-	90	60	35	15	0	-40	-	-	-	-
	0.5	-	-	80	45	20	0	-45	-90	-	-	-
	0.6	-	-	-	60	25	0	-55	-105	-	-	-
	0.7	-	-	-	-	40	0	-70	-125	-	-	-
3.0	0.2	40	30	20	15	5	0	-15	-25	-	-	-
	0.3	55	40	25	15	5	0	-15	-30	-45	-	-
	0.4	-	50	30	20	10	0	-20	-35	-50	-65	-
	0.5	-	-	-	25	10	0	-20	-40	-55	-70	-
	0.6	-	-	-	-	15	0	-25	-45	-60	-80	-
	0.7	-	-	-	-	-	0	-30	-55	-75	-90	-
5.0	0.2	20	15	10	5	5	0	- 5	-15	-20	-20	-
	0.3	-	20	15	10	5	0	-10	-15	-20	-25	-
	0.4	-	-	15	10	5	0	-10	-15	-20	-25	-45
	0.5	-	-	-	10	5	0	-10	-20	-25	-30	-50
	0.6	-	-	-	-	-	0	-10	-20	-30	-35	-55
	0.7	-	-	-	-	-	0	-15	-25	-35	-40	-60
7.0	0.2	15	10	5	5	0	0	- 5	-10	-10	-15	-25
	0.3	-	15	10	5	0	0	- 5	-10	-10	-15	-25
	0.4	-	-	-	5	5	0	- 5	-10	-15	-15	-25
	0.5	-	-	-	-	5	0	- 5	-10	-15	-20	-30
	0.6	-	-	-	-	-	0	-10	-15	-20	-20	-35
	0.7	-	-	-	-	-	0	-10	-15	-20	-25	-35

For $p_r = p_s = 0.10$ the correction is $g_2/10$ (rounded down).

Conversion factor f_1 for N due to a change in w_2.

Reference value of w_2 = 0.05, $p_s = p_r$.

Use $N^* = Nf_1$ as argument in the master table to find (n^*, c^*).

$100w_2$	1.5	2.0	p_2/p_r 3.0	5.0	7.0	p_1/p_r
1	0.54	0.56	0.58	0.61	0.63	0.2
	0.46	0.48	0.51	0.54	0.57	0.7
2	0.70	0.72	0.74	0.76	0.77	0.2
	0.65	0.66	0.68	0.71	0.73	0.7
3	0.82	0.83	0.84	0.86	0.87	0.2
	0.78	0.79	0.81	0.83	0.84	0.7
4	0.92	0.92	0.93	0.94	0.94	0.2
	0.90	0.90	0.91	0.92	0.93	0.7
5	1.00	1.00	1.00	1.00	1.00	0.2
	1.00	1.00	1.00	1.00	1.00	0.7
6	1.07	1.07	1.06	1.06	1.05	0.2
	1.09	1.09	1.08	1.07	1.06	0.7
7	1.14	1.13	1.12	1.10	1.09	0.2
	1.17	1.16	1.15	1.13	1.12	0.7
8	1.19	1.18	1.16	1.15	1.13	0.2
	1.25	1.24	1.21	1.19	1.17	0.7
9	1.25	1.23	1.21	1.18	1.17	0.2
	1.32	1.30	1.27	1.24	1.22	0.7
10	1.30	1.27	1.25	1.22	1.20	0.2
	1.39	1.37	1.33	1.29	1.26	0.7
12	1.38	1.36	1.32	1.28	1.25	0.2
	1.51	1.48	1.43	1.38	1.34	0.7
14	1.46	1.43	1.38	1.33	1.30	0.2
	1.63	1.58	1.52	1.45	1.40	0.7
16	1.53	1.49	1.44	1.38	1.34	0.2
	1.73	1.68	1.61	1.52	1.46	0.7
18	1.60	1.55	1.49	1.42	1.38	0.2
	1.83	1.77	1.68	1.58	1.52	0.7
20	1.65	1.60	1.53	1.46	1.41	0.2
	1.92	1.85	1.75	1.64	1.56	0.7

Correction g_1 to n^* due to a change in w_2.

Reference value of $w_2 = 0.05$, $p_s = p_r = 0.01$, $n = n^* + g_1$.

	p_2/p_r					
$100w_2$	1.5	2.0	3.0	5.0	7.0	p_1/p_r
1	-125	-90	-60	-35	-25	0.2
	-205	-125	-70	-35	-25	0.7
2	-70	-50	-35	-20	-15	0.2
	-115	-70	-40	-20	-15	0.7
3	-40	-30	-20	-10	-10	0.2
	-65	-40	-25	-10	-10	0.7
4	-20	-15	-10	-5	-5	0.2
	-30	-20	-10	-5	-5	0.7
5	0	0	0	0	0	0.2
	0	0	0	0	0	0.7
6	15	10	5	5	5	0.2
	25	15	10	5	5	0.7
7	25	20	15	5	5	0.2
	45	25	15	10	5	0.7
8	40	30	20	10	5	0.2
	60	40	20	10	10	0.7
9	50	35	20	15	10	0.2
	80	50	25	15	10	0.7
10	55	40	25	15	10	0.2
	90	55	30	15	10	0.7
12	75	50	35	20	15	0.2
	120	70	40	20	15	0.7
14	85	60	40	25	15	0.2
	140	85	50	25	15	0.7
16	100	70	45	25	20	0.2
	160	100	55	30	20	0.7
18	110	80	50	30	20	0.2
	175	110	60	30	20	0.7
20	120	85	55	30	20	0.2
	195	120	65	35	25	0.7

For $p_s = p_r = 0.10$ the correction is $g_1/10$ (rounded down).

Summary of conversion formulas

to find

(n,c) corresponding to $(N,\ p_r,\ p_s,\ p_1,\ p_2,\ w_2)$

from

(n^*,c^*) in the master table for $(N^*,\ p_{ro},\ p_{1o},\ p_{2o})$.

For $\left\{\begin{array}{lll} p_r & \leq & 0.05 \\ p_r & > & 0.05 \end{array}\right\}$ use master table with $p_{ro} = \left\{\begin{array}{l} 0.01 \\ 0.10 \end{array}\right.$.

$$\lambda_s = \left(1 + \frac{p_s - p_r}{w_1(p_r - p_1)}\right)^{-1} .$$

Formula 1.

$$\gamma_2 = \frac{w_2(p_2 - p_r)}{w_1(p_r - p_1)} \quad \text{and} \quad \lambda p_{ro} = \frac{p_2 + 19\gamma_2 p_1}{1 + 19\gamma_2} .$$

Use

$$N^* = N\lambda_s\lambda,\ p_{ro},\ \ p_{1o} = p_1/\lambda,\ \ p_{2o} = p_2/\lambda$$

as arguments to find (n^*,c^*) in the master table.

$$(n,c) = (n^*/\lambda, c^*).$$

If (p_{1o},p_{2o}) fall outside the tabulated range use formula 2.

Formula 2.

$$\lambda = p_r/p_{ro},\quad \rho_1 = p_1/p_r,\quad \rho_2 = p_2/p_r.$$

Use

$$N^* = N\lambda_s\lambda f_1(w_2,\rho_1,\rho_2),\ p_{ro},\ \ p_{1o} = \rho_1 p_{ro},\ \ p_{2o} = \rho_2 p_{ro}$$

as arguments to find (n^*,c^*) in the master table.

$$(n,c) = ((n^* + g_1(w_2,\rho_1,\rho_2))/\lambda, c^*).$$

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